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MANUAL

OF

ELEMENTARY LOGIC

DESIGNED ESPECIALLY FOR THE USE OF TEACHERS AND LEARNERS.

BY

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PREFACE.

THE object of this volume is to furnish a text-book of instruction for the use of teachers and students of Elementary Logic. This object has determined its contents and form. It does not claim to offer any new contribution to the science of Logic, as such, although it is quite possible that, in some instances, the author's way of illustrating known truths may have shed some new light upon them. Still less is it designed to present an exhaustive treatise containing all the truths pertaining to Logic which have been reached by the great masters and expounders of the science.

But, as before stated, the object is to present the great elements of the science in a form suited to the wants of teacher and learner. Books for this purpose of decided merit are indeed now in use. Many of them, however, are not constructed in conformity to the now recognized conception of Logic, as the Science of the Laws of Thought. Others are too extended, cumbrous, or abstruse, for elementary instruction, especially within any time that can possibly be allotted to this study in our Colleges and High-schools. Some of them need much previous drill in more elementary treatises as a propædeutic.

At all events, the author in this brief manual has attempted to meet a want which has become urgent in his own personal experience as a teacher. How far it meets the wants of others remains to be seen. He can only bespeak for it a candid judgment and fair trial.

It is only just to add that he has freely used whatever best served his purpose in the works of other authors, sometimes without explicit mention of the sources from which he has drawn. It is proper, however, to say that he is in greater or less degrees indebted to the works of Whateley, Kant, Hamilton, Mansel, Bayne, De Morgan, Wilson, Bowen—and most of all to Thomson's *Laws of Thought*. He trusts this general acknowledgment will suffice for all cases in which none more specific is made.

Perhaps no better place will occur for stating, that occasional paragraphs will occur of such a description that, though important, they may be postponed for review, or omitted entirely, if pressure for time requires. It is of course always the teacher's province to judge how far any portions of the several chapters may be wisely postponed until the time of review. The author will, however, suggest that Sections IV. and V., of Chapter I., may advantageously be deferred until the student reaches Chapter VI., when it will form a suitable introduction to the study of applied Logic. The beginner can better understand and appreciate it at this point, than in that natural order in which it is treated in defining the sphere and objects of logical science. The same view applies in a less degree to Section VII. of the same chapter, on the Uses of Logic.

Portions of this will of course be better understood after the student has learned somewhat of the principles involved. On the other hand, it is a strong reason for giving early attention to it, that some idea of the advantages of the study is a strong stimulus to the student to make the effort necessary for its successful prosecution.

NOTE TO REVISED EDITION.

While such minute corrections of the main text of this treatise as have been found needful will be found in this edition of it, the chief improvement consists in the large addition of examples for logical praxis which so greatly facilitate the teaching and study of Logic.

PRINCETON COLLEGE, 1879.

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ELEMENTARY LOGIC.

CHAPTER I.

THE SPHERE AND OBJECTS OF LOGIC.

SECTION I.

1. LOGIC IS THE SCIENCE OF THE LAWS OF THOUGHT OR THINKING. Logic defined.

2. Of these two words, thought and thinking, we shall hereafter use the former to denote the object matter of Logic. Thought may denote either the process or the product of thinking, *i. e.*, it may be taken either in a subjective or objective sense. Logic is the science of the laws of thought in both senses; the laws which govern genuine thinking itself, and also the relations of the products of thought to each other, and to all matters to which they are applicable. Thinking or thought, subjective and objective.

3. *Object* means that about which the mind thinks; *Subject*, the mind itself. The adjectives *subjective* and *objective*, and the Subject and Object defined.

adverbs *subjectively and objectively*, have a corresponding import; the former in each case referring to the mind considered as the *subject* of conscious states of knowing, thinking, feeling, willing; the latter referring to whatever becomes an object of the mind's attention. And since the mind may make itself, its own states and exercises, objects of its attention, it is said, in this case to *objectize* itself, or become a *subject-object*. When it is needful to discriminate other objects from this *subject-object*, some writers use the term object-object. The student who understands the foregoing, will easily understand the terms *objectively* and *subjectively*, when they come in his way. The sooner these terms are understood, the better, as they are of constant occurrence, not only in philosophy, but in general literature.

Subject-object.

Object-object.

4. The next step in clearing the subject is to determine what thought is.

Thought defined as the working and product of the Discursive Faculties.

Thought is *subjectively* the operation, and *objectively* the product of the Discursive Faculties of the mind.

5. It becomes necessary now, in order to make this definition complete and intelligible, to explain what we mean by the Discursive Faculties. Although

this is properly within the province of Psychology, yet it is at one of those points of contact between it and Logic, which requires to be explained in defining the object-matter of either.

The Discursive Faculties explained.

6. For our present purpose then, the faculties of intelligence, (leaving out of view memory which retains and reproduces what is given by the other faculties), may be divided into two great classes—the Intuitive and the Discursive.

Twofold division of Intellectual Powers.

7. The Intuitive Faculties are those which discern objects, phenomena, or presentations immediately, and not indirectly, *i. e.*, not through the medium of any process of thinking. Thus, the objects perceived by the senses are known intuitively, as whatever we see, hear, touch, taste, or smell. So also our states of consciousness, our feelings, volitions, cognitions, at the moment of their occurrence, are known intuitively. The mind knows them immediately, *intuetur*, by a direct beholding, and without the intervention of reasoning or thinking.

Intuitive Faculties described.

8. The Intuitive Faculties furnish us the original material of all our know-

They furnish the material of Thought.

ledge. The Discursive Faculties take the matter thus furnished, and proceed from it, *discurrunt*, to new results founded upon it. They work it up or elaborate it into new forms; hence by Hamilton and others, they are called the Elaborative Faculties.

The Discursive elaborate it into new forms.

Why called Elaborative.

9. It is important to observe, that intuitions (aside of exceptions in the region of self-evident supersensual truths), that is to say, intuitions of material things or of states of consciousness, are always of individual objects, never of classes of objects. By the senses we perceive individual trees, stones, or animals. But the senses do not apprehend them in classes. To classify is to perform a process of Abstraction and Generalization, *i. e.* of Thought, and goes beyond intuition. So of states or acts of consciousness. They are first perceived singly, not in classes. Now this pure intuition is not thought strictly so-called, nor in the sense here intended. It furnishes matter for thought, but is not thought. With this logic does not concern itself, unless casually and indirectly. It develops the laws of the think-

Intuitions are of individual objects.

Further intellectual processes are discursive.

Logic concerns the latter, not the former.

ing process, and of its products, in their constituent parts, combinations, and relations.

10. The Discursive Faculties are those which take the materials furnished by intuition, and, by a process of thought, involving Analysis and Synthesis, reach new results. First, they separate or analyze the single objects or wholes given in intuition into parts. They notice one or more of the parts into which any individual whole is thus analyzed, to the exclusion of the residue. That is, they abstract them from the rest. Thus, suppose that this book be the object beheld. It has extension, figure, solidity, color, is composed of printed sheets, enclosed in binding, and is a treatise on Logic, etc. Now the mind may attend to one or some of these properties, neglecting the rest. This is ABSTRACTION.

Discursive Faculties proceed by Analysis and Synthesis to new results.

Abstraction described.

11. Again, the mind, observing a number of objects that agree in one or more particulars singled out by abstraction, forms a class or genus of objects which so agree. Thus, noting extension, not only in this book, but in every material object, it classifies them as extended objects. Observing that, besides extension, they have

Generalization illustrated and explained.

solidity, it forms them into the genus, bodies or matter. Noting also that many of these agree in being composed of sheets of paper for the purpose of containing written or printed language, it classifies them as such, under the name of books. This is the manner in which it forms *genera* or classes from individual objects. And to do this is to *generalize.* It is obvious, moreover, that generalization may proceed, not only from individuals to classes, but from lower genera to higher, which comprehend them: as from white-oak, yellow-oak, scrub-oak, live-oak, to oak; and from oak, hickory, ash, etc., to tree; and from tree, grass, flower, grain, etc., to vegetable; and so on, till we arrive at the highest possible generalization (*summum genus*), which is Being. Hence Logic treats first,

OF CONCEPTIONS.*

12. The product of this Generalization is conception (*con capio*) the taking of many together in

* This is the meaning to which logicians now limit the word conception, viz., *that act or product of the mind which is denoted by a general term, and is obtained by generalization.* In common speech, it has a much broader import, and is used almost synonymously with that loosest of words, idea, *i. e.* for almost any mental act or representation. And by philosophers it has been used

one, *i. e.* in one class, denoted by one name. This conception or the name denoting it, represents, not all of any individual object, but so much thereof as is common to it with the whole class of which it is one. Thus the conception *bright* denotes, not all of any one bright object, but so much of it, as it has in common with all bright objects.

Conception explained.

13. This conception, or mental representation of what is common to a plurality of objects, may be abstract, or viewed by itself irrespective of any objects to which it belongs, as brightness; or concrete, *i. e.* belonging to some object, as bright moon.

Concrete and Abstract Conceptions.

14. It may also be considered subjectively and objectively, either with reference to the mental pro-

almost as vaguely. Particularly they have used it to denote the mental similitudes of past cognitions or objects of cognition which are raised in the mind by the exercise of memory. As, when I remember a house, I have a mental image, or as these philosophers would say, a conception of it. So of the products of Constructive Imagination—new combinations, which are not mere copies or images of any thing else. These, too, by many authors, of whom Reid is an eminent example, are styled conceptions. The strict scientific use of the term, however, in present philosophic nomenclature, is to signify the mental exercise or product of generalization.

cess forming it, or with reference to the product of that process, considered as formed, and made the object of our thinking. Some writers limit the word "conception" (*conceptio*) to the former; and denote the latter by the word "concept" (*conceptus*). And as logic, in evolving the laws of this product of thought, makes it the object of attention, these writers use the word "concept" exclusively to denote this, which is the primary element within this sphere of this science. Since, however, this word serves no purpose not equally well accomplished by the word "conception," we shall adopt the latter to denote the first object-matter that falls within the sphere of the science of logic, *i. e.* the products of Abstraction and Generalization; of which, be it observed, in passing, the former may take place without the latter, but not the latter without the former.

Subjective and Objective Conception. Concept.

Generalization involves Abstraction.

15. Conceptions, and, indeed, the whole process of generalization, are incomplete, fugitive, and unavailable, until they are set, and so to speak, encased and preserved in names. Each one may easily test this for himself, by an examination of his own consciousness. He

Conceptions incomplete without names.

will see that he cannot retain, or employ, to any extent, in judgments and reasonings, the ideas or conceptions denoted by general words, without the words themselves. The attempt to preserve and turn to account our generalizations without naming them, has well been likened to the process of making conquests, and leaving them without fortifications for their security and preservation.

16. Hence, as terms are so implicated with the conceptions for which they stand, we may often use the two interchangeably. The older logicians were wont more commonly to use the former when treating of this department of their science. Some, of whom Whateley is a prominent example, have carried this view to the extreme of maintaining that Logic is wholly conversant about language. This has been pronounced by others, as Hamilton, to be utterly groundless. The truth is, assuredly, that logic is primarily and properly conversant about thought, and about language incidentally as the vehicle of thought. The science of language is Grammar, or Philology, and not Logic, which is the science of the laws of thought.

Terms and Conceptions interchangeable.

Extreme views.

17. And yet, owing to the inseparable connec-

tion, amounting, for practical purposes, to almost an identification of thought and language, there is a sense obviously, in which Whateley's doctrine may be regarded as a half truth—often the worst form of error.*

Sense in which Whateley's doctrine is true.

18. The first part of Logic then has to do with that product of thought which results from generalization, called Conception; and with terms or names incidentally, as being the vehicles of conceptions.

The first part of Logic deals with Conceptions or Terms.

The next of the Discursive Faculties is Judgment. And it gives as its products the second great object of logical science, to which we now proceed.

Judgment.

Sect. II. Logical Judgments.

19. We say Logical Judgments, because there is a sense in which judgment is a constituent of every act of mind or exercise of consciousness. If we have a pain we cannot but judge that we have it. Consciousness is the knowledge of our mental operations, and inseparable from them. Of course, the knowledge

Logical and Primitive Judgment compared.

* This interpenetration of thought and language may go far to reconcile and clear up the dispute between the Nominalists and Conceptualists.

that we have them, is in some sense, a judgment that we have them. For distinction's sake this, which enters into all the intuitions of the mind, may be called Primitive Judgment. It furnishes the materials out of which conceptions and logical judgments are ultimately framed. The only predicate which it gives is that of existence. It simply affirms that a given phenomenon external or internal *is*.

Primitive Judgment; its only predicate existence.

20. Logical Judgment, on the other hand, includes a conception as one, or conceptions as both, of its elements. It compares two conceptions, or a conception and an intuition, and affirms that they agree or disagree. Thus it affirms of the conception "man" and the conception "rational," that they agree, *i. e.* that "man is rational." So likewise of horse and quadruped, tree and plant, etc., etc. Or if we take an individual object of intuition named Pompey, and the conception man, or horse, as the case may be, we may affirm that "Pompey is a man;" "Pompey is a horse." And negatively, we may affirm that the conceptions man and quadruped do not agree; "man is not a quadruped;" that the particular object called Pompey

Logical Judgment defined.

Compares Conceptions or Conceptions and Intuitions.

and the conception, philosopher, do not agree. Pompey is not a philosopher. Similar examples of all these forms of judgments the reader can easily multiply at his pleasure.

21. Remarking here provisionally, that a judgment consists of two parts or terms (termini, extremes) the Subject, or that which is spoken of, and the Predicate or that which is said of the subject, it follows from this definition, that while the subject may be either an intuition or conception, the predicate must always be a conception or common term, the name of a class. If we have Peter for the subject, unless we have a common term as predicate, we can get only the senseless tautological judgment, Peter is Peter. Of judgments it is unnecessary now to say more, in marking out the sphere of Logic, than that they constitute the second great product of thought, and object of Logic as the science of the laws of thought.

Terms, Subject and Predicate defined.

The Predicate always a conception.

22. From Judgments the mind proceeds to derive other judgments founded upon them. This is Reasoning, or inference from premises to conclusion. Thus to conclude from premises is in fact to

The mind proceeds from judgments to new judgments founded upon them.

judge. So all modes of thought, from conceptions to reasonings are in reality forms of judgment. The third and last great province of Logic, therefore, is the laws of reasoning.

All thought in reality a form of judgment.

Sect. III. Reasoning.

23. This runs into various branches or modes, Mediate and Immediate, Categorical and Hypothetical, which need not be further defined nor explained till we come to treat of it in form and in length.

The third province of Logic is Reasoning.

Until a recent period, it was largely the custom of logicians to treat Reasoning as constituting the whole primary object-matter of their science, and to bring Judgments and Conceptions, under the name of Propositions and Terms, into the sphere of Logic, only on the ground of their being elements of the Syllogism and other forms of reasoning. But they invariably treated of these terms and judgments in many aspects of the first importance, which are not immediately essential to the Syllogism, or other forms of Reasoning. Thus Whateley has a short introductory chapter in explanation of terms (conceptions), so far as their relation to forms of reason-

Former place of Reasoning in Logical Treatises.

ing is concerned, while he postpones the consideration of them in chief, till he has finished the analysis of the various forms of the Syllogism. This shows that these conceptions and judgments have a separate and independent place in Logic on their own account, and in their own right, irrespective of their place in the Syllogism. This will be more evident when the student reaches these subjects. Indeed, it is only necessary to think of Genus, Species, Differentia, Essence, Accident, Absolute, Relative, Correlative, etc., as applicable to Conceptions, to see that these have in their own right, a leading place in the science of Logic. The definition of Logic, till recently in vogue, as being the science of Reasoning, is therefore too narrow. It is, as we have defined it, and as the present masters of the science generally define it,

Conceptions and Judgments have a place in Logic in their own right.

Recapitulation.

I. THE SCIENCE OF THE LAWS OF THOUGHT.

Definition of Logic.

II. THOUGHT IS THE OPERATION, OR PRODUCT OF THE OPERATION, OF THE DISCURSIVE FACULTIES, AS DISTINGUISHED FROM THE INTUITIVE.

Of Thought.

III. The Discursive Faculties are,

a. Abstraction and Generalization; the product of which is Conception.

b. Judgment, which out of Conceptions forms Logical Judgments.

Enumeration of Discursive Faculties.

c. Reasoning which from judgments given evolves other judgments founded upon them.

The thinking and products of thinking, whose laws Logic unfolds, therefore, are, Conceptions, Logical Judgments, Reasonings.*

Logic deals with Conceptions, Logical Judgments, and Reasonings.

* I also rank Constructive Imagination among the Discursive Faculties. Its operations and products, therefore, are of the nature of thought. As we unfold the laws of thought, it will appear that they cannot be violated, even in the creative works of this faculty. They may be violated in the apparent form, sound, and sense of the language employed, and the imagery constructed; but not in its real interior significance. All appearance of thought which violates these laws, is not genuine thought, but a counterfeit or simulation of it. The creations of imagination cannot abolish the laws of Conception, Judgment, Reasoning. They cannot legitimate contradictions, render a round-square possible or conceivable, or make arguing in a circle valid. If it tells us that rain-drops are the tears of the sky, it means such resemblance between the tears and rain-drops as actually exists. The laws of Logic, therefore, so far as applicable to Constructive Imagination, are developed in treating of Conceptions, Judgments, and Reasonings.

Sect. IV.—Pure and Applied Logic.*

24. Having defined the sphere of Logic, and pointed out the matters with which it deals, it remains that we further elucidate it, by showing what it is in itself considered as pure science, in distinction from the application of its principles to the investigation of truth and the ascertainment of facts —Pure and Applied Logic.

Pure Logic deals with the laws of Thought irrespective of their Applications.

Pure Logic treats of the Laws of Thought as they are in themselves, whatever be the object-matter to which they are applied, and irrespective of their application to any case of actual being. Its principles and laws, like those of Pure Mathematics, are true in themselves, irrespective of their application to cases of actual being, nay, whether there be any actual being to which they are applicable or not. The laws of the Syllogism, the conditions of valid reasoning, the principles which determine genus, species, differentia, essence, logical division and definition, are the same, whatever be

* This and the following chapter may be passed with advantage for the present, to be taken up as an introduction to Chapter VI. on Applied Logic.

the objects to which they are applied, whether angels, men, animals, plants, or grains of sand; and aside of such applications.

In this Logic classes with Mathematics, and with strict Metaphysics. The rules of Arithmetic, and the propositions of Geometry are true, irrespective of their applications to actual being, and in respect to whatever kinds of actual being furnish the conditions to which they are applicable. The Multiplication table is true in itself, irrespective of any actual being, and in regard to all actual being to which it is applicable. $12 \times 12 = 144$. This of itself, however, does not prove any truth of actual being. It does not prove that there are twelve persons, each twelve years old. But it does prove, that if there are twelve such persons, their aggregate age is 144 years. Logic, as such, does not concern itself with the original sources of our knowledge of actual being, or of the conditions to which it applies. These may be supplied by intuition, or testimony, or legitimate logical deduction from them. They may, in various aspects, come within the province of Psychology, Metaphysics, Ontology, or the different

It classes with pure Mathematics and Metaphysics.

Does not in itself give original knowledge of actual being.

departments of physical science. But from whatever sources the requisite conditions of actual being are furnished, to which any of the principles of Logic apply, the corresponding consequence necessarily follows. Logic does not prove that gold is fusible, or that gold is a metal; but given these truths from whatever source, and it follows that some metal is fusible, on principles of Logic.

It is a science of necessary truths. "Necessary" defined.

25. Hence, pure Logic, like pure Mathematics, is a science of *necessary* principles or truths. By necessary we mean that the opposite of which, the mind cannot conceive to be true without intellectual suicide. Such are the following, "that the whole is greater than a part," that "all qualities must belong to some substance," that "no two straight lines can enclose a space." So, as in the proper place the student will more fully see, that there can be no valid conclusions in a syllogism vitiated by negative premises, illicit process, or undistributed middle; that every relative supposes a correlative, that we may predicate of a species its genus and differentia; these, with all other laws of pure Logic, are necessary truths. They are not only true in particular cases, but, when understood, it is seen that

they *must* be true, as the rules of Arithmetic and the propositions of Euclid must be true in all cases. Hence pure Logic is not only, as before shown, the science of the laws, BUT OF THE NECESSARY LAWS OF THOUGHT.

Logic the science of the necessary laws of Thought.

26. This characteristic classes Pure Logic with the *a priori*, as distinguished from the *a posteriori* sciences. By *a priori* knowledge is meant that which is known from conditions given, without needing verification from experience. *A posteriori* knowledge depends upon experience for proof. The axioms and propositions of Geometry are *a priori*, because they are known and proved independently of experience. The physical and inductive sciences, on the other hand, are *a posteriori*, because they are dependent on experience for proof. Hence, all sciences of necessary truth, including Logic, are *a priori*, for they not only show what experience *has* proved true; but what ever *must* be true in all possible experience, and must condition that experience. We know *a priori*, that no two straight lines *can* enclose a space, and that every equiangular triangle *must* be equilateral. So we

Pure Logic an a priori science.

Definition of a priori and a posteriori.

know, as the student in the proper place will see, that, as the Extension of a conception increases, its Intension must diminish, and vice versa: and that there can be no conclusion from negative premises.

27. It is putting the same thing in another light, to say that the laws developed by Logic, are those which are necessary to the very *form* of thinking, whatever be the subject-matter about which we think, and independently of such subject-matter. The forms of thinking in Conceptions, Judgments, and Reasonings, are the same, whether applied to planets or to worms; just as the forms of Arithmetical Addition, Subtraction, &c., are the same, to whatever they may be applied: and the opposite sides of a parallelogram are equal whether it be on wood, slate, iron, or between lines imagined in pure space. This truth is set forth by saying that Logic is the science of the forms of thought; or of the formal laws of thought—either phrase will serve our purpose sufficiently well. And so combining all the elements thus far shown to be comprised in the essence of Logic, we reach this definition: PURE LOGIC IS THE SCIENCE OF THE NECESSARY AND FORMAL LAWS OF THOUGHT.

Logic deals with the Forms of Thinking.

Completed definition of Logic.

Those sciences, the Mathematics, Logic, and, within certain limits, Metaphysics, which deal with truths, not within themselves originally implying actual being, but which are forms regulative of such actual being as presents the conditions to which they apply, are called Formal Sciences. Those on the other hand which have what, in these relations, is called *Content*, or matter of actual being, whether in the realms of body or spirit, are called Material Sciences. The contrast here is not between Material and Spiritual, but between Material and Formal. The opposite of Spiritual is Physical Science. Matter and Material in these connections refer to substances and phenomena of actual being, whether bodies or spirits. Accordingly, pure Logic is one of the Formal Sciences.

Other Formal Sciences.

Form, Content Matter.

Material Sciences.

How related to Spiritual, also to Physical Sciences.

28. These are also sometimes named Hypothetical Sciences; because they prove truths of actual being only on the *hypothesis*, that the conditions of actual being are given to which they are applicable. Thus, that the angle in a semi-circle is a right-angle proves no

Hypothetical Sciences explained.

fact of actual being, until we have some substances in the form of a semi-circle, with an angle inscribed in it. Such an angle we know *must* be a right angle.

Sect. V.—Applied Logic.

29. In the actual investigation of truth, we must go beyond Pure Logic, which, of itself, like Mathematics, deals only with forms of thought, and has no content of actual being. Yet, like Mathematics, it is of the utmost value as an instrument or calculus in the investigation of truth. The primary facts, which lie at the basis of astronomical science, were not obtained by mathematics but by telescopic observation. Mathematics is an instrument for determining what is fairly involved in, or results from these facts so observed. By its use the former science has made the immense strides which have advanced it to its present perfection. So Geometry and Trigonometry will not of themselves make a science or art of Navigation, Surveying, or Engineering. They cannot furnish the facts which underlie these sciences. But the application of these Mathematics to facts otherwise

Pure Logic a calculus.

Application of Formal Sciences to facts a means of discovering truths.

discovered, is indispensable in these sciences, and alone makes them possible.

30. So is it with the laws of Thought unfolded by Logic. They do not, of themselves, prove any original fact of existence; but, given such data, as are furnished by other means, it is an instrument for showing what is and what is not fairly contained in them: for unfolding explicitly what is involved implicitly: for guarding us against unwarranted conclusions from given facts or truths; for guiding us to the avoidance of fruitless, and the adoption of fruitful methods of inquiry in the realms of actual being. Such use of the principles of Logic in assisting us to right, and preserving us from wrong processes of thought in our search after truth, is what is meant by APPLIED LOGIC. This has two departments.

Uses of Logic.

Applied Logic defined.

31. *a.* The doctrine of FALLACIES. Showing the various ways in which men consciously or unconsciously employ, a mimicry of thought, especially of reasoning, for the things themselves, thus sometimes imposing upon themselves, or essaying to impose on others.

Its two departments. Fallacies and Method.

b The doctrine of METHOD, or the right way to ascertain the truth, by modes of investigation, not contrary to, but harmonious with the laws of Thought.

32. Pure Logic then treats of the formal and necessary laws of Thought in Conceptions, Judgments, and Reasonings. Applied Logic deals with the application of these laws to the detection of Fallacies, and the development of a proper Method for the investigation of Truth. Before proceeding, however, to the formal consideration of each of these topics, we will make a few preliminary observations, first on the utility of the study of Logic, and secondly on the fundamental principles or axioms of the science.

Summation.

SECT. VI.—UTILITY OF LOGICAL STUDY.

33. The study of Logic is useful as a means of disciplining and invigorating the mind. Few studies more effectually promote habits of attention, discrimination, and continuous application.

Uses of Logic. Intellectual Discipline.

34. The knowledge thus acquired is of high value on its own account. All knowledge is precious and elevating; but es-

Imparts valuable knowledge.

pecially that which sheds light on the laws of our thinking, our intelligent and rational nature.

35. It is invaluable as furnishing the nomenclature, the Technical Terms, which define the products and relations of true Thought, and the nature of the fallacies which counterfeit it. The possession of these names in a multitude of cases will instantly suggest to the mind the clew to difficulties which would otherwise perplex it. The very terms, *genus, differentia, petitio principii, ignoratio elenchi, arguing in a circle,* will of themselves often suggest an analysis or explanation of perplexities which otherwise might long be insoluble.

Furnishes apt Technical Terms.

36. Generally, as a guide to right, and a preventive and corrective of spurious thinking, *i. e.* of the aimless, erratic, and abortive exercise of our faculties. So it is a propædeutic to all other sciences. It furnishes a needful training for every department of study. So it has been crowned by some, as *scientia scientiarum,* by others, as *ars artium.*

Guide to right Thinking.

37. The question has been much discussed whether Logic is a Science or an Art. But as the end of Science is to know, and of

Logic a Science, and Guide to Art.

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Art to do, or rather to make a product which survives the making, so there can be no doubt that pure Logic is, like pure Mathematics, properly a Science; while Applied Logic, like Applied Mathematics, may afford great light in the learning and executing of the arts to which it is applicable, as the art of Reasoning, Rhetoric, and Oratory. Although not useful as in itself an art, it is useful as furnishing light and guidance in the noblest arts.

It gives Psychology of Discursive Faculties.

38. The study of Logic as the science of the Laws of Thought, gives, in fact, if not in form, the knowledge of Psychology, so far as the faculties of Thought are concerned. Although the necessary and formal laws which all true Thought *must* obey, are not of themselves psychological phenomena, yet it is impossible to master them, in their application to the phenomena of the Discursive Faculties, without so far forth understanding the psychology of those faculties. So far as Abstraction, Generalization, Conception, Judgment, Reasoning, are concerned, little remains to be learned, which is not acquired in a thorough course in Logic, in the present acknowledged scope of that science. It is easy for the teacher, with little addition of labor, to compass this portion of psycho-

logy, in connection with his regular course in Logic —a matter of some moment, in view of the scanty time generally allowed to those subjects.

39. It has indeed been said that men reason, whether they know Logic or not. They are not dependent on Logic to confer on them the power of reasoning. Even Locke is guilty of such poor burlesque on this high subject, as the following. "God has not been so sparing to men to make them barely two-legged creatures, and left to Aristotle to make them rational. . . . God has been more bountiful than so; He has given them a mind that can reason without being instructed in methods of syllogizing," etc.* This is quite as relevant, as if one should say, "God has not been so sparing of gifts to men, as to leave it merely to the grammarians to confer the gift of speech, or to the rhetoricians to confer the gift of composition and oratory." The science of Grammar, of course, does not confer the gift of speech. It presupposes that gift. But that it helps to the correct use of language, who will dispute? Rhetoric does not first

Objections of Locke and others refuted.

Analogy of Grammar and Rhetoric.

* Quoted by Whateley—*Logic.* Harper's Edition, p. 37.

make men eloquent; but who can doubt that, rightly used, it will greatly augment this gift of eloquence in those naturally endowed with it? Logic does not impart the power of reasoning or thinking. But who will question that it greatly assists in detecting and avoiding the spurious counterfeits of them; and that it is every way a great intellectual tonic? Locke is not alone, even among men of mark in philosophy and literature, in this vulgar and Vandal disparagement of Logic, which, if admissible against this, is valid against all liberal study, discipline, and culture. No less a man than Macaulay has allowed himself to indulge in reflections and implications of like force and effect in regard to Grammar and Rhetoric as well as Logic.*

Macaulay.

SECT. VII. FUNDAMENTAL PRINCIPLES OR AXIOMS OF LOGIC, FROM WHICH ALL ITS PARTICULAR LAWS FLOW, OR BY WHICH THEY MAY BE TESTED.

40. These are commonly reduced to the four following — IDENTITY, CONTRADICTION, EXCLUDED MIDDLE, AND SUFFICIENT REASON.

The Four Fundamental Principles.

* See *Essay on Lord Bacon.*

I. The principle of IDENTITY, which amounts simply to this: that we may affirm of objects that they are what they are. This lies at the foundation of all Positive Conceptions, and Affirmative Judgments, and Reasonings. Thus if the Conception rational be a part of the Conception man, we may affirm that "man is rational." On the same ground, we may have the Conception "rational animal," because these may concur in the same being.

Identity.

II. CONTRADICTION. That is we may not affirm the co-existence of Conceptions or attributes that are mutually contradictory, as "round-square," "triangular parallelogram," "good wickedness."

Contradiction.

III. Of two contradictories one must be true, and the other false. There can be no medium between these. This is the Law of EXCLUDED MIDDLE.

Excluded Middle.

IV. For every conclusion, affirmation, or negation, there must be a SUFFICIENT REASON OR GROUND. It must be evinced by self-evidence, or other sufficient evidence.

Sufficient Reason.

41. These principles may seem too obvious and familiar to be the foundation of any important science. But we must bear in mind, that the highest sciences are but developments from a few simple elements or axioms. The science of Mathematics is but a development or evolution of a few axioms as simple as the foregoing. Herein, very largely, lies its adamantine strength. What are the laws which keep the myriads of orbs harmoniously circling in the depths of space, but developments and applications of the simple but great law of gravitation? And does not the highest of authorities teach us that, on the simple obligation to love God with all the heart, mind, soul, and strength, and our neighbor as ourselves, "hang all the law and the prophets?" That is, that all the details of religion and morals, are but the logical unfoldings of this simple principle?

Importance of these principles.

CHAPTER II.

Section I.—Conceptions.

1. In unfolding the nature of Conceptions, as also, of Judgment and Reasoning, it will be necessary occasionally to repeat a few things, which were unavoidably introduced by way of anticipation in our brief preliminary exposition.

2. Conceptions stand contrasted with Intuitions, which cognize single presentations, whether external or internal, whether bodies or states of consciousness, immediately and intuitively. Conceptions, on the other hand, grasp (con-capio) a plurality in one, through the medium of a common sign or mark, whereby they are, so far forth, represented. This plurality may be of objects thus brought to unity in a common genus, by a common mark or resembling quality, as the whole class of red things are brought

Conception and Intuition compared.

Conception grasps a plurality in one.

This plurality may be either of objects or marks, included under a common name.

to unity, or classified by the common mark of redness. Or it may be a plurality of marks or attributes under one name. As *hexagon* includes the two marks, rectilineal figure and six sides.

Expressed by a General Word.

Another aspect of the same truth is, *Conception is that act or product of the mind which is expressed by a General Word.* And hence,

Formal Definition of Conception.

3. *Conception is that product of the mind which results from Generalization, whereby many individuals are combined in one class, through one or more similar qualities, and are indicated by a common term.* Thus, certain pieces of iron-ore are observed to have the property of attracting iron, and are generalized into one class under the name Magnets. It is obvious that, in attending to this quality of attracting iron, exclusively of others, there is a withdrawing or abstracting it from them. Here is Abstraction. There is Comparison, in order to detect the resemblance of these qualities in the several magnets. Then there is the Classification or Generalization by virtue of this resemblance. Finally, in order to complete and guard the product of this process, the name "Magnet" is applied to this class.

Involves Abstraction.

Comparison.

Generalization.

This is Denomination. Thus we have a conception formed as the result of Abstraction, Comparison, Generalization, Denomination. Denomination.

4. NOTION is a term of wider import than Conception. It is used almost as looseiy as Idea. It includes representations not only of Conception, but of mental similitudes of objects remembered by simple Imagination. Notion. Idea.

5. Conceptions and the corresponding terms which express them, may be viewed either as,

ABSTRACT,

i. e. as expressing a quality irrespective of any object in which it inheres, as Magnetism, Heat, Wisdom, Virtue. Or they may be viewed as, Abstract Conceptions.

CONCRETE,

i. e. as inhering in some object, as magnet, hot-blood, wise man, virtuous person. These distinctions will also apply to the inherence of higher in lower conceptions, as will be seen when we come to define this distinction. They also prepare us to understand the distinction between Dentative, Connotative, and Non-Connotative terms. Concrete.

6. A Term is Denotative in so far as it denotes an object or objects. All names of single objects, *i. e.* Singular Terms, have this capacity, whether they be proper names, or common terms with an individualizing particle; as "John," "this man." All strictly concrete terms, as fools, stones, trees, have this capacity, besides their power to connote. Abstract conceptions have not this capacity. They include qualities but not objects, as virtue, color, wisdom.

Denotative Terms. Singular Terms.

7. Connotative (which are also Attributive), terms or conceptions *de*note objects, and *con*note qualities along with them, as men, roses, animals. Such are all Adjectives, inasmuch as they express qualities belonging to the objects indicated by the names to which they belong. The Adjectives foolish, organized, etc., can only be used in reference to their appropriate objects. When, however, adjectives are used to qualify abstract nouns, they denote not so much objects, as the quality which they still further determine. Thus, "*great* virtue," "*scrupulous* veracity." Of course, all concrete common nouns, as horses, quadrupeds, etc., are connotative. They *de*note objects and *con*note qualities.

Connotative Attributive.

8. Non-connotative words are proper nouns which denote objects simply; also abstract common terms, which denote qualities (and in this sense have denotative power), but *con*note no objects; as blackness, harmony, etc. Proper names ordinarily denote intuitions or single objects, not conceptions. Non-Connotative.

9. Proper Names sometimes acquire the attributes of common terms, when the individuals they denote become types of a class. As when we speak of a Webster, a Washington, a Napoleon, or of the Cæsars and Nimrods of our race; *i. e.* the class of men who have the qualities of Cæsar or Nimrod. In such cases, these names are connotative. Adjectives formed from them are like other adjectives in this respect, as British subjects, a Websterian or Johnsonian style, *i. e.* a style having the qualities of the style of these authors. Proper Names become common.

Sect. II.—Higher and Lower Conceptions.

10. It is evident that the same process of generalization may be applied to classes as to individuals. Thus triangles, squares, parallelograms, polygons, etc., may all Generalization of classes as well as individuals.

be generalized into the one class of rectilinear figures. Circles, ellipses, parabolas, etc., may be reduced to the one class, curvilinear figures. Rectilinear and curvilinear again may be united as one in the higher genus, plane figure. Dogs, lions, horses, etc., may be generalized into the higher class of quadrupeds. And so of numberless examples which will readily occur to the student. Now in such cases, the broader conception which includes the others, is called the Higher. The narrower ones which are included, are the Lower. Quadruped is a higher conception than dog or fox. As the process of combining lower conceptions into a higher, by laying aside their differences, is Generalization; so that of resolving the higher into the lower, by adding on these differences, is called Determination. The Conception triangle undergoes this process when it is resolved or determined into equilateral, isosceles, and scalene.

Higher and lower Conceptions.

Determination of Conceptions.

11. In the scale of higher and lower Conceptions we have another application of the distinction of the Concrete and the Abstract. A higher Conception which is Abstract when taken by itself alone, becomes Con-

Concrete and Abstract applied to Classes.

crete when incorporated with another in a lower Conception. Thus the Conception rationality is Abstract, when taken by itself alone, but when united with animality it becomes Concrete in the lower Conception manhood.

Sect. III.—Genus, Species, Individual, Differentia, Essence, Accident, Property.

12. In any series of higher and lower Conceptions, each higher is a Genus to those next below it, out of which it is formed by generalization. Those next below it are its Species. Thus birds, fishes, beasts, reptiles, men, are species to the Genus animal. Differentia, or Specific Difference, is the mark or quality which distinguishes one species from others under the same Genus. Individual or Intuition is that which is logically indivisible, although it may be capable of physical division. It cannot, therefore, be a species, although it may be one of the constituents of a species. An ox cannot be divided logically, but may be physically into hide, horns, quarters, etc. But then it is no longer an ox. Of course then an individual can

Genus.

Species.

Differentia or Specific Difference.

Individual.

never be a Species or Genus, which is always composed of a plurality of individuals. Essence refers to Species, and its essential constituents, *i. e.* its Genus and Differentia. These are called Essence, because when present the Species is present; if either be absent that is wanting. These which constitute the Essence of a Species, also constitute Logical or Essential Definition. As rose (Genus), red (Differentia), constitute the Essence or Definition of red-rose. Accident, or Accidental Conception belongs to a part, and not to the whole of a class, as sickness or health to man. Property belongs to the whole of a Species, but is not a part of its Essence: as liability to laugh, or grow gray, in man whose Essence is (Genus) animal, (Differentia) rational. Where these are, whatever else is wanting, there is manhood. Where they, or either of them, are not, there manhood is not.

Essence is Genus and Differentia.

Logical Definition.

Accident.

Sect. IV.—Subaltern and Proximate Genera and Species. Summum Genus and Infima Species.

13. In a series of higher and lower Conceptions, it has been shown that the same one may be a Genus to those next below, and Species to that next above.

Those Species to which any given Species becomes a genus, are relatively to it Subaltern Species. Those Genera which are Species of a higher Genus are called Subaltern Genera. Thus White-oak, Yellow-oak, Live-oak, etc., are Subaltern Species to oak, which is a Species of the genus tree; and is therefore a Subaltern Genus to it. *Summum Genus* is that highest class which is never a species. *Infima Species* is that lowest class which is never a Genus.

Subaltern Genus and Species.

Summum Genus. Infima Species.

14. Proximate Genera and Species are those which are next to each other in order of ascent or descent. Thus triangle is the Genus proximate to equilateral, isosceles and scalene triangle. They are proximate Species of triangle.

Proximate Genera and Species.

15. It should be noted that *Summum Genus* may be Absolute, with reference to the Universe, in which case it is Thing or Being simply; or it may be Relative to a particular department—as animal is Summum Genus of corporeal beings having life and consciousness: plane superficial figure with reference to triangle, square, etc. And it is sometimes fixed arbitrarily with reference to the purposes of some parti-

Absolute Summum Genus. Relative also.

Summum Genus and Infima Species often arbitrarily fixed.

cular discussion. *Infima Species* is also often difficult to be fixed, for it is often hard to find classes that have no sub-classes. It might be supposed that isosceles triangle was Infima Species among plane superficial figures. Yet it may be divided into those of different magnitudes: and each of these again into those drawn on slates, boards, paper, etc. This therefore is seldom reached absolutely. It is rather fixed somewhat arbitrarily with reference to the exigencies of the inquiry in hand.

16. It is important to note the difference between Species in Logic and in Natural History. In Logic, as has been shown, it means one of the proximate lower classes into which any higher class or genus may be divided. The same class may thus be Genus to a lower, and Species to a higher.

Logical and Natural Species distinguished.

In Natural History, however, Species means only such a class of animals as has, or might have descended from a single pair, and the varieties of which may permanently inter-propagate among themselves. These sub-species are by the Naturalists rigidly named Varieties. Bull-dog, terrier, grey-hound, etc., are Varieties of the Species, dog.

Varieties.

In a Logical sense, quadrupeds, reptiles, birds, fishes, are species of the genus animal. In the Naturalistic sense, though they include Species, they are not themselves Species at all, as they want the marks already noted, of actual or possible descent from a single pair, and of inter-propagation. We are aware that some naturalists adopt other criteria of natural species. This, however, is not the place for extended discussion of that question.

Sect. V.—The Three Powers of Conception. Extension, Intension, and Denomination.

17. From the analysis already given of the formation of Conceptions, it appears that they include a plurality of objects through their resembling qualities indicated by a common name, and that the number of objects so included, increases with the height of the Conception. Thus man includes more objects than poet, orator, philosopher; and animal more than man. This power to denote objects constitutes the Extension of Conceptions. Extension of Conceptions.

It is equally plain that every conception includes or connotes qualities or marks. The ground of classification is resem- Intension or Comprehension.

bling qualities. Therefore the conception of any class involves these similar qualities or marks which constitute it. Thus the conception square involves the following marks: 1. Rectilineal figure: 2. Having four sides: 3. And those sides equal: 4. And its angles right angles. The conception man involves the marks, 1. Animal, 2. Rational. This power of conceptions constitutes their Intension, formerly called their Comprehension, which by Whateley has been identified with Extension.*

Denomination.

It is not less clear that conceptions have the capacity to receive names, and must receive them in order to be preserved and used. A conception without a name, is like an unfenced crop, or a volatile odor. This is the power of Denomination.

To these three powers of Conception, three important processes respectively correspond, viz.: Division to Extension; Definition to Intension; and Explanation to Naming or Denomination.

* See Logic, Harper's Edition, p. 152.

Sect. VI.—Inverse Ratio of Extension and Intension, or Comprehension.

18 As the Extension of Conceptions increases, their Intension diminishes. It is by laying aside the distinctive marks of lower conceptions, that we rise to higher, that is, more extensive conceptions.

As Extension increases, Intension diminishes.

Thus by laying aside the distinctive marks, Equilateral, Isosceles, and Scalene, we arrive at the higher conception, Triangle, which has greater extension, and less intension than isosceles, or scalene triangle. So poet, orator, statesman, have less extension and greater intension than man. The ratio of these to each other, therefore, is inverse. Conceptions then may be regarded as embracing or constituting the respective wholes of Extension and Intension, each of which decreases as the other increases. Of course in Summum Genus Extension reaches its maximum, and Intension its minimum; and conversely in Infima Species. These wholes have sometimes been called respectively, the former Logical, the latter Metaphysical. We agree, however, with Hamilton, that this distinction is without any suffi-

Examples.

Logical and Metaphysical Whole.

cient ground, each alike being, in one aspect Logical, and in another Metaphysical.*

Sect. VII.—Denomination.

19. The process of Denomination keeps pace alike with the Extension and Intension of Conceptions. Thus, as the extension is increased, names are employed to denote each enlarged class, till we reach the highest, which is Being or Thing. And vice versa; as we add on successive marks to Being, names are applied to include or connote them, till the term man includes being, with life, sensation, and reason. All this is well illustrated in the following tabular examples from *Thomson's Laws of Thought*, which we copy, because it is hard to find or invent any other, in all respects so much to the purpose.

Names keep pace with the Extension and Intension of Conceptions.

* Various other modes of expressing this double capacity of a Conception are in vogue. Thus a Conception viewed as an

Extensive Whole,	Intensive Whole.
has	has
Extension,	Intension or Comprehension,
Breadth,	Depth,
Sphere,	Matter,
Objects,	Marks,
Power to Denote,	Power to Connote.

"*Scheme of* CONCEPTIONS *in the three wholes of* EXTENSION, INTENSION, *and* DENOMINATION.

	A. Name.	B. Intension, *i. e.* the *marks* which compose the Conception.	C. Extension, *i. e.* the *objects* which the Conception stands for.
(Process of	Denomination.	Abstraction.	Generalization.)
Genus — Summum	*Body*	Body	Stone, Plant, Brute, Man, etc.
Genus — Subalternum	*Living Body*	Body with life	Plant, Brute, Man.
Genus — Subalternum	*Animal*	Body with life and sensation	Brute, Man.
Species — Infima	*Man*	Body with life, sensation, and reason	Man."

This table has value, not only as illustrating the mutual relation logically, of Intension, Extension, and Denomination, according to the exposition of them which has been given, but also as affording a valuable scientific classification of objects which it concerns all to know.

SECT. VIII.—VARIETIES AND CHARACTERISTICS OF CONCEPTIONS USUALLY EXHIBITED WITH RESPECT TO THEIR DENOMINATION, OR THE NAMES WHICH COMPLETE AND INDICATE THEM.

Various kinds of Terms marking corresponding features of Conceptions.

20. As Conceptions are incomplete till they are named, and these names are called Terms, or Nouns, so certain features of Conceptions are usually set forth as belonging to these terms or nouns. But as these terms stand for Conceptions, so the different kinds of Terms are but different kinds of Conceptions, save in those exceptional cases of proper names, in which they denote only intuitions or individuals.

Proper, Common, Singular.

21. The first division of nouns is into Proper, Common, and Singular. Proper names denote individuals merely, without connoting any marks or qualities. Common names denote conceptions, and the objects included in them, together with their common marks, *i. e.* their extension and intension. Singular terms denote single objects by means of a common noun, having its signification limited by an individualizing particle, as this man, a house, some animal.

22. The second principal division of nouns is into Attributive and Substantive. Attributives are the adjectives of Grammar. They express qualities not in the Abstract, but in the Concrete, as belonging to some substance. They express the attributes of Nouns, and are therefore used only in connection with the Nouns of which they are adjuncts. Substantive Nouns denote objects or abstract qualities, to which Attributives may be applied. Thus tree and hardness are Nouns Substantive. High and great are attributives which may be respectively ascribed to them.

Attributive.

Substantive.

23. Another distinction is that between Distributive and Collective Nouns. A Noun is Distributive or used distributively, when it is applicable to each and every individual included under it. It is Collective, or used collectively, when it is applicable to the whole, or a plurality only, but not to each and singular of the objects included under it. Thus man is a Distributive, and crowd a Collective Noun. Soldier is a Distributive, army a Collective Noun. The same noun may, however, be used both collectively and distributively. When we say, "these trees are oaks," trees are used distributively. When we

Distributive.

Collective.

say, "these trees amply shade this park," they are used collectively. Hence a Term is said to be Distributed, when it is so used as to include all the objects signified by it distributively; that is, all and singular of them, not merely a part of them, nor the whole of them collectively. When we say, "all men are mortal," men is Distributed. If we say, "some men are poets," men is not Distributed; and if we say "all men number 1,300,000,000," men is not Distributed: for although all men are spoken of, it is not all and singular, but all taken collectively, that are meant.

Distribution of Terms.

Terms or Conceptions are Absolute and Relative. Absolute are irrespective of any other, as stone, tree. Relative are those which imply others. As son implies a parent, and king a subject. A pair of relatives like father and son are called Correlatives.

Absolute and Relative.

Correlatives.

In all Relative Conceptions there is a ground of the relation (*fundamentum relationis*). In the case of king and subject, it is government. In that of father and son, brother, sister, etc., it is the family. Some relatives imply not merely one, but two, or even several Correlatives. Thus, cousin implies not

Ground of Relation.

Cases of several Correlatives.

only another cousin, but parents, one of whom is brother or sister of one of the parents of the other cousin.

24. *Contrary and Contradictory Terms or Conceptions.* Contraries are the most opposed that can possibly belong to the same subject, as wise and foolish, soft and hard. Contradictories are simple Negatives of each other, and between them include all being actual and possible. Thus, man and not man, Ego and non-Ego, are pairs, each of which comprises the universe, not only of actual, but of possible being. And of such a pair of Conceptions one only marks out any definite class of objects. They are for this reason called Definite and Indefinite Conceptions. Of the two Conceptions, man and not-man, the former alone contains any thing definite or positive either as respects objects or qualities. The latter is not only indefinite, but essentially infinite. It embraces all the possibles but man, the subtraction of which does not make their number less than infinite. Hence such purely Negative Conceptions are sometimes classed by logicians as Infinitated Conceptions.

Contraries.

Contradictories

Definite and Indefinite.

Infinitated Conceptions.

25. It is not, however, true of most Negative Conceptions, that in their real and customary significance, they have this infinity. Especially is this not true of Attributives. Thus, if we speak of unkindness, we do not mean every thing that is not kind, but we mean the absence of this quality in intelligent and moral beings who ought to be kind, and in whom to be unkind is to be harsh or severe. Now a conception or term which implies the presence of any mark is called Positive, as virtue, wisdom, benevolent. A term which implies the absence of what *might* belong to a given subject, is Privative, as an unkind or unholy man. Negative terms on the other hand, deny not only what does not, but what cannot belong to some given object, as lifeless stone, speechless block. These do not belong to the class of Infinitated Conceptions.

Most Negatives not really Indefinites or Infinites.

Positive.

Privative.

Negative.

26. These distinctions are not without practical importance. In the first place they add to our variety of forms of thought and expression, and so to the means of precision of style. The words unkind, unholy, unlearned, give us shades of thought not expressed by

Importance of these Distinctions.

the words harsh, wicked, ignorant. Again, the distinction of Privative and Positive is of moment, in reference to the origin of evil as related to God. He is in no sense the cause of sin, except privatively or negatively. It may arise from the absence, not the presence, of his agency, as darkness arises not from the presence but the absence of the sun.

27. Two terms which may be applied to the same object at the same time, are called Compatible or Consistent, as red and round to a table; diligent and healthy to man. They are Opposite or Inconsistent when they cannot be applied simultaneously to the same object, as "round square figure," "lifeless breathing man."

Compatible and Consistent.

Opposite and Inconsistent.

28. The important distinctions of Abstract and Concrete, Connotative and Non-connotative terms, were sufficiently explained when treating of the corresponding conceptions. To these the student can recur, chap. II., sect. I. 5, and II. 11.

Sect. IX.—Quality of Conceptions.

29. By the Quality of Conceptions is meant the degree of perfection with which they represent to the mind the objects and

Quality of Conceptions defined.

the marks included in them. In this regard, Conceptions, like all cognitions, are perfect in proportion as they have the several virtues of Clearness, Distinctness, and Adequacy. In proportion as they have the opposite vices, they are respectively Obscure, Confused, and Inadequate. The nature of these respective virtues and faults we will now proceed to explain.

When they are Perfect.

30. A conception or other cognition is Clear, when it is simply distinguishable from others, and Obscure when it is not. Thus in twilight we often see objects, but are unable to distinguish them from each other. Our cognitions of them are obscure. As the light gradually comes upon them, our view becomes so clear that we can distinguish them apart. The uninstructed cannot distinguish Logic from Psychology and Metaphysics, or a Court of Chancery from a Court of Law. These are Obscure conceptions to those unversed in such matters. All persons are afflicted with more or less of this obscurity of knowledge in departments to which they have not given special attention.

Clear and Obscure.

31. But we may know objects or conceptions, so as to distinguish them from each other, without

being able to point out the marks by which they are so distinguished. Such knowledge may be sure as far as it goes. But it is confused with respect to the marks or differential features of the object. This is among the most common phenomena of our intelligence. How common to know persons of our acquaintance from each other, without being able to specify the peculiarities of form or feature which distinguish them severally. How common to be sure as to the hand-writing of different persons, without being able accurately to define the peculiarities of each. How often do lawyers in court perplex witnesses, and torture out of them absurd answers, by asking them the marks by which they identify the persons or the hand-writing in regard to which they testify. Yet what tribunal ever discredited a witness on account of any puzzle or inconsistency into which he was thus drawn? Those, however, who have made such subjects a study, are able to give the marks of difference. Their knowledge is distinct, while the other is confused. The same distinction holds in regard to our understanding of conceptions. If we take the conceptions mineral, plant, animal, man, how few who

Distinct and Confused Cognitions explained.

Illustrations.

do not surely know the one from the other? But how few can accurately give the marks which distinguish them respectively from each other? A Clear cognition or conception then knows its objects from other objects. An Obscure one does not. A Distinct cognition or conception not only knows its objects, but the marks of those objects. A Confused one knows its objects without knowing their marks.

Distinction of Distinct and Confused.

32. This Distinctness of our conceptions may be both Adequate and Inadequate. It is Adequate when it not only apprehends their marks, but the marks of these marks. And when it fails of this, it is Inadequate. Thus we have a Clear knowledge of the conception man, when we discriminate it from animal, plant, etc. We have a Distinct knowledge of it, when we know its marks to be animality and rationality. This knowledge is Adequate when we can give not only these marks of manhood, but can also give the marks or definitions of animality and rationality, those of the former being life and sensation, of the latter the intuition of supersensual truths and the power of thinking in the light of these truths. This process of giving the marks of marks is in itself capable of indefinite

Adequate and Inadequate.

extension. That measure of it which is adequate, cannot be decided by any unvarying rule. It varies with the exigencies and requirements of each particular discussion, and must often be determined somewnat arbitrarily.

No Unvarying Rules of Adequacy.

SECT. X.—NOTATIVE AND SYMBOLICAL CONCEPTIONS.

33. This is a pregnant distinction. A Notative Conception is such that when presented to the mind, it suggests its own marks (*notæ*) by its very name, so that they are at once and indubitably evident, *e. g.* quadruped, triangle, octagon, oligarchy. A Symbolical Conception is one which serves as a symbol of a number of marks or characteristics which it does not, of itself, bring before the mind using it. It is used as a substitute for, or representative of, the marks which the mind does not stop to bring in detail before itself, and, indeed, which, in many cases, it could not, if it would. Such are the conceptions or terms, church, family, senate, philosophy, etc. Few bring before their minds all the marks involved in these conceptions. Most persons could not do it, who, nevertheless always

Notative Conception.

Symbolical Conception.

use them with substantial accuracy. All conceptions which are used without an apprehension of their marks or definition, are used Symbolically.

34. All Thorough Knowledge is obtained by removing from our conceptions the several imperfections of Obscurity, Confusion and Inadequacy, and developing these into Clearness, Distinctness, Adequacy and Particularity; as also by unfolding the marks of Symbolical Conceptions till they have something of the distinctness of Notative Conceptions. This is no less essential to invention and style in Rhetoric, than to logical thinking.

Thorough Knowledge how obtained.

It is accomplished by two great processes, each of which must be pursued in proportion as we would make our conceptions clear, distinct, and adequate.

The first of these is Logical Division, which unfolds the Extension of Conceptions.

The second is Definition, which unfolds their Intension.

These processes are now to be considered.

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Sect. XI.—Logical Division.

35. Logical Division divides a Genus according to its extension, *i. e.* into its constituent and proximate Species. It may then take any of these proximate Species for a Genus, and divide that into sub-Species. In like manner it may divide any of these again, and so on, until we pass through Infima Species to individuals.

Logical Division Defined.

36. The Genus divided as being the higher, is sometimes called the Super-ordinate. The proximate species into which it is divided, are called Co-ordinates. If either of these be divided into parts or Species, with reference to its superior Genus, it is called Subordinate Genus. Any one of given Co-ordinate Species, is called, in relation to any one part of a higher or lower Co-ordinate Division under the Summum Genus, Disparate. Thus, quadruped is super-ordinate to lions, leopards, horses, cats, etc. They are co-ordinate with each other. They are subordinate to quadruped and animal, while lion, as compared to fish, Shetland pony, or bull-dog, is Disparate.

Super-ordinate Genus.

Co-ordinate Species.

Subordinate Genus.

Disparate Species.

Examples.

37. The rules for correct Logical Division are:

a. It must proceed from Proximate Genera to Proximate Species, and not *per saltum*, or arbitrarily. To Divide animals at once into whales, sturgeon, etc., without previously dividing them into birds, fishes, etc., would be a violation of this rule.

Must be from Proximate Genera to Proximate Species.

b. There must be but one principle of Division, *fundamentum divisionis*. In dividing a library, for example, it will not do to divide the books according to price and according to binding, at the same time. To do this is to violate the

But one Principle of Division.

c. Third Rule, which is, that the Divisions must be mutually exclusive. They must not run into each other by cross-divisions. This will result from adopting more than one principle of Division. Thus if we divide the books of a library according to their subject-matter, and according to the language in which they are written, some books of poetry, history, and oratory, will be in Latin, French, English, etc. One fruitful source of perplexity and confusion in the discussion of subjects is unobserved cross-divisions, which ought rigorously to be avoided.

Divisions Mutually Exclusive.

d. All the parts should be exactly equal to the genus divided; any one part, and the sum of any number of parts less than all, should be less than the Divisum or genus divided. To divide mankind into rational animals and all others, or into Europeans, Asiatics, Americans, and Greenlanders, would be a violation of this, as well as of other rules.

Sum of parts equals Divisum.

e. It must not be *a priori*, or by Infinitation. For this, although in form regular and exhaustive, is in fact useless. It adds nothing to our knowledge. To divide animals into partridges and all others, or partridges and not-partridges, is indeed a formally complete, but a completely useless Division. Such a Division into two members, which inevitably are contradictories, is called a Dichotomy. A division in three members is called a Trichotomy: into many members, a Polytomy.

Not a priori.

38. PHYSICAL DIVISION or PARTITION.—Logical Division must be clearly distinguished from Physical Division or Partition. The latter divides an individual, which is logically indivisible, into its component parts, as a ship into hull, masts, sails, etc. The test of this

Individuals not logically divisible.

sort of Division is that the Divisum cannot be predicated of parts. In Logical Division it always may. We cannot predicate ship of sails, masts, etc., but we can predicate it of steamship, sailing-ship, etc.

It should be further observed, that we may arbitrarily make collective wholes logical individuals, when it suits the end in view. Thus nations, armies, regiments, etc., may be treated as Logical individuals. Often like literal individuals they cannot be so divided that the divisum can be predicated of the parts. Thus army cannot be predicated of regiments, nor regiments of companies, nor nations of towns.

Collective Wholes Logical Individuals.

39. The thorough logical division of any subject, thus defining the sphere and the objects it includes greatly assists the clear, thorough, and facile discussion of it. It also aids invention. The most sterile mind will find something to say on a subject well mapped out. Indeed so to map it out, is to say something important. Division gives clearness to our Conceptions by pointing out their objects. But to gain distinctness and adequacy, we must resort to

Uses of Division.

Sect. XII. Definition.

40. This gives the marks of Conceptions, and unfolds their Intension. It thus bounds them off from all other Conceptions, so that we not only know that they differ, but in what way they differ. The rules for correct definition are,

Definition described.

a. It must be by essential marks. The essential marks of a species are what constitute its essence, *i. e.* its genus or matter, and differentia or form. This is normal, logical definition, or definition strictly so-called. All other definition is valid in proportion as it approximates to this.

By essential Marks.

☞ *Let not the student forget when asked what is logical or essential definition, that it is made up of the genus and differentia.*

Logical Definition.

b. It must include the objects covered by the definitum, or species defined, neither more nor less. If it include more, it is too broad. Thus to define a whale as a fish, is too broad. To define a fish as a whale is too narrow. A definition too broad is detected by simple conversion. If it is a good definition of a whale to say that

Not too Broad nor Narrow.

it is a fish, then all fish are whales. A definition too narrow is detected by conversion by contraposition. Thus, if it be a good definition of a fish, that it is a whale, then whatsoever is not a whale is not a fish. For the fuller understanding of this, the student must recur to it after studying the subject of conversion, in its proper place, under the head of Reasoning.

How Detected.

c. It must not be by Negatives, if this can be avoided. Negatives show what are not, instead of what are marks, and so add little to our knowledge. To define man, as not an angel, or not a brute, is unsatisfactory. It does not tell what he is. There are, however, Negative words and conceptions, which in their very nature require a negative definition, as unholy is simply not holy.

Not by Negatives.

d. It must not be in vague, ambiguous, or senseless language. To say that "truth is the grand scope of all existence," or that "beauty is the harmony of being," are examples in point.

Must be in clear Language.

e. It must not be Tautological, *i. e.* through the word defined, or any of its derivatives, or synonyms from other tongues, or the

Not Tautological.

negative of its opposites. To define life as the vital force, or the state of living, or the opposite of death, is thus to err. This is definition in a circle, *circulus in definiendo*, for such definitions return upon themselves. If light be defined as "that which illuminates," per contra, "that which illuminates is light." The circle in definition as in argument, is often unobserved. How easy to define a plank as a thick board, and a board as a thin plank?

f. It must be Precise and free from surplus words. These surplus words, though true, may convey a false implication. To say that a parallelogram is a rectilineal four-sided figure, whose opposite sides are parallel *and equal*, is to state the truth. But the words "and equal," are unnecessary to the definition: and they convey this false implication that there may be such figures whose opposite sides are parallel, but not equal. This vice is of more frequent occurrence in ordinary thought and speech, than in formal definition. How natural to say, "we ought not to calumniate so good a man," as if it were right to calumniate anybody?

Precise and free from Surplusage.

41. Absolute Summum Genus cannot be logically

defined, because it has no differentia. Thus, being can only be defined by some synonym or description casually substituted for it. It may be defined as a thing, or that which has existence. Summum Genus in any particular sphere, being such only relatively, and always a species of a higher genus, is of course capable of strict logical definition.

Absolute Summum Genus not Logically Definable.

42. Simple Ideas are incapable of logical definition, as they cannot be analyzed into elements, and therefore are without genus and differentia. They can only be defined like Summum Genus, by synonymous or descriptive equivalents. Red is a color. This is genus. But who can give the differentia, that separates it from other colors? What is color? What is goodness or beauty? What is the respective genus and differentia of each? But although not definable, do they need defining? Are they not self-evident and plainer in themselves than any definition could make them?

Simple ideas likewise.

43. Logical definition, strictly considered, refers only to Species, and therefore does not technically apply to Individuals. Hence other methods of defining them have been de-

Individuals, how Defined.

vised. They may be defined by the intuition of them, through the senses if they be bodies, or through the light of consciousness, if they be mental states. Or they may be defined by some peculiar and inseparable Accidents, as a virtual Differentia. Thus, Cicero might be defined as "the greatest Roman Orator," and the first Napoleon as "the greatest French General." They are, however, thus defined by all that is essential in a logical definition. They are referred to the Infima Species under which they fall, and discriminated from other individuals under it, by some mark peculiar to themselves. Thus Washington may be defined as "the first President of the United States." Here the Infima Species, President of the United States, is to the individuals under it, what every proximate genus is to its co-ordinate species. This then may be taken as the genus, and "first" as the differentia.

44. Indeed, in all definition, whether of individuals or species, the genus and differentia may be considered as two communicant genera, and the Conception defined that which is included within the sphere of their coincidence. Either may be considered

Genus and Differentia really two Communicant Genera.

genus and the other differentia, and vice versa, at convenience. Thus, if we define a man as a rational animal, this extends over so much of the conceptions, rational and animal, as overlap each other. Thus:

In like manner, so much of the genera, "Presidents of the United States," and "fourth," as overlap each other, are just equal to, and define James Madison, fourth President of the United States.

45. As there are many cases in which a strictly logical definition is either impracticable or inconvenient, several other methods of defining are occasionally adopted, which serve more or less effectually to clear the definitum, and to bound it off from all else. Including these, the methods of definition in all amount to six, arranged by logicians as follows:

Methods of Definition.

a. Resolution. This resolves the Conception into its marks, genus and differentia, and is,

Resolution.

as we have seen, the standard, normal, logical, essential definition. Thus, "man is a rational animal."

b. Composition. This is the reverse of resolution, and unites the marks into the Conception of which they are concrete parts. Thus, "a rational animal is man." Composition.

c. Division: i. e. according to extension, into its constituent parts, whether species or individuals. Thus, "the New England States are Maine, New Hampshire, Vermont, Massachusetts, Connecticut, and Rhode Island." "The animal kingdom consists of Radiates, Mollusks, Articulates, and Vertebrates." Division.

d. By *Colligation,* the reverse of the last, *i. e.* uniting the constituent parts acccording to extension together, as James, John, Matthew, Thomas, etc., were the twelve Apostles. The Earth, Mars, Mercury, Venus, etc., are the Planets. This formula furnishes the minor premise for the Inductive Syllogism. Colligation.

e. By the substitution of Symbols or names; as "religion is piety." Exchange of Symbols.

f. By Casual Substitution of narrative or descriptive phrases, as, *wisdom leads to virtue and happiness.* This last, how- Casual Substitution.

ever, hardly comes up to the exactness required in real definition. Nor should the fifth method be used when it can be avoided.

46. Most logicians refer to the distinction between Nominal and Real Definition, and strangely inconsistent accounts have been given of it. These terms are adapted to mislead. Nominal Definition is the Definition of a name; Real, of a thing. But the Definition of a name is none the less a Real Definition. Indeed, all strictly Logical Definitions are of names, and give the marks which these names stand for. That is, they give the marks connoted by these names. This is the proper, normal province of Definition. As to qualities of things not connoted by the name, they are important, and belong to scientific investigation and the increase of our knowledge, but do not directly constitute Definition. They may afford the means of correcting or improving the accepted Definition of these names, which is Definition proper. In Mathematics and the Ideal and Formal Sciences, the Definition of the Name, is of necessity the Definition of the Thing. The Definition of the names, "Circle," "Conception," "Extension," "In-

Nominal and real Definition.

tension," etc., is, of course, the Definition of the thing.

47. From this analysis, it appears that Definition, or the distinct explication of the marks of conceptions, is of fundamental importance in thinking, investigating, and discoursing. An accurate Definition, or presentation of the *status questionis*, will often settle controversies otherwise interminable. Without such Definition, all discussion and investigation must be futile and unsatisfactory. And it is quite as powerful a stimulus to invention as Logical Division.

Importance of Definition.

48. It is, moreover, quite plain that Definition and Division are mutual helps to each other. When we Divide a Genus into its proximate species, we, of necessity, are bringing to light the differences between those species. These, with the Genus, make up the Definition. On the other hand, looking for the differences, we, of course, are finding the boundaries of the several species into which Division separates the genus.

Definition and Division mutual aids.

CHAPTER III.

JUDGMENT.

Section I. Its Constituent Parts.

1. JUDGMENT is that act of the mind which, upon comparing two Conceptions, or an individual object of intuition with a Conception, affirms that they agree or disagree; that they do or do not belong to each other. Thus, "Victoria is queen." "Angels are not men." A Judgment expressed in words is a Proposition. Judgments and Propositions are always either true or false. No other form of thought or expression has these attributes.

Judgment defined.

Proposition.

2. Strictly speaking, as has been already observed, in the last analysis, every intelligent act is a Judgment. To know is to discriminate, and therefore to judge. Even feeling and sensation, the most rudimental form

Strictly every Mental Act a Judgment.

of consciousness, involves a knowledge and so a Judgment that it exists. This is Primitive as distinguished from Logical Judgment. And yet it is hard to maintain this distinction without qualification. For the most Primitive Judgment affirms that something is or is not; *i. e.* it affirms that the Conception, Existence, agrees or disagrees with some subject. But beyond the mere predication of Existence, Primitive Judgments do not go. Logical Judgments are founded on Conceptions formed by Abstraction and Generalization from these Primitive Judgments. Yet, since Primitive Judgments involve the Conception of Existence, which withal is Summum Genus, the two flow into each other.

Primitive Judgment.

Predicates existence only.

Logical Judgments.

3. And it is to be observed, that all the processes of Thought, whether by Conceptions, Judgments, or Reasonings, in reality proceed from and terminate in Judgments. Conception is the product of the Judgments involved in abstraction and generalization, whereby many objects, through some common mark or point of similitude, are grasped together. Conception fixes and preserves this Judgment, by a common name.

All Thought resolvable into Judgments.

Thus, the conception and the name, bi-ped, is the fruit and confirmatory sign of the Judgment, that animals, which agree in having two feet, may be put in a class denoted by the common name, biped. As Conception is the product of a Judgment, so it is explicated by a Judgment. Thus "bi-peds are two-footed animals." "Animals are conscious." In short, Conception and Judgment interpenetrate each other. In one view, Conception is a certain stage of Judgment. Judgment in form develops Conception. Reasoning, too, the third great process or form of thought, deals only with Judgments, and their relations to each other, as will be seen, when we come to treat of it. It proceeds from one or more Judgments given to others founded upon them. Thus, in the last analysis, Logic being the Science of Thought is the Science of Judgments, into which all thought is finally resolvable. Nevertheless it is convenient to treat of pure formal Judgment by itself, after Conception which furnishes the materials of Judgments; and before Reasoning, which is composed of them, and

Conceptions formed and explicated by Judgments.

They interpenetrate each other.

Reasoning also is by Judgments.

Logic the Science of Judgments.

in concluding that one Judgment flows from others, forms the Judgment that it does so.

4. Judgment being thus a mental affirmation of the agreement or disagreement of two notions, one of which at least is a Conception, these two notions are called the terms (*termini*, extremes) of the Judgment. Terms of a Judgment.

That which is spoken of is the Subject of the Judgment. That which is affirmed or denied of the other is called the Predicate. That which connects the two is the Copula. This is always the verb to be, in the Present Tense Indicative, if the Judgment be affirmative: and the same with the negative particle affixed, if the Judgment be negative. Thus: Subject, Predicate. Copula is verb "to be" in present tense.

Sub.	Cop.	Pred.
The earth	is	round.
Sub.	Cop.	Pred.
Oaks	are not	pines.

The Copula, in many cases, is not directly expressed by the word *is*, or *is not*, but is in other phrase, which implies them. When any other than the Substantive verb is employed as Predicate it includes the Copula. Thus, Copula often implied.

"he runs," is equivalent to, he is running. "No men are sinless," is the same as to say of all men that hey are not sinless.

When Existence simply is expressed, the verb to be is both Predicate and Copula; as, God is = is existing.

Predicate when implied.

5. When any mood or tense of the verb, except the present indicative in the Copula, is significant, this significance belongs to the Predicate and not to the Copula. Thus, if we say, "This farm *was* fertile, whether it be so now or not," it is the same as to say, this farm is one formerly fertile. The weather *may* be good, the weather is what may be good. As either term of a Judgment may be a Conception including different objects, or having several marks, so several words may be employed to make up a term. Thus, "The dews of the evening are the tears of the sky." "Birds, fishes, beasts, and reptiles, are animals."

What belongs to the Predicate.

6. Words which alone cannot express conceptions, or intuitions, cannot of themselves constitute terms of a Judgment. They can

Categorematic and Syncategorematic Words.

only enter into these terms by combination with verbs and nouns substantive and adjective. Such are articles, prepositions, conjunctions and adverbs. These are Syncategorematic; nouns, adjectives, and verbs, on the other hand, are Categorematic, because they can of themselves be Terms.

Sect. II.—Quantity, Quality, Relation, and Modality of Judgments.

7. Judgments may be viewed, I. With reference to the relation of the predicate to the extension of the subject—Quantity. Judgments in respect of Quantity.

II. With respect to the relation of the predicate to the intension of the subject—Quality. Quality.

III. With respect to the manner of connecting the predicate with the subject—Relation. Relation.

IV. With respect to the degree and kind of certainty in the connection of subject and predicate—Modality. Modality.

Sect. III.—Quantity of Judgments.

8. With respect to Quantity, Judgments are either Universal, Particular, or Singular. Judgments are Universal when the Pre- Universal Judgments.

dicate is affirmed or denied of all the Subject taken distributively, as, "all men are sinners;" "no men are angels."

Judgments are Particular when the Predicate is affirmed or denied of an indefinite part of the subject, as, "some men are orators;" "some Governments are not Democratic."

Particular.

Judgments are Singular; *a.* when the Predicate is affirmed or denied of individuals, as, "Cæsar was a Conqueror;" "this man is not learned." *b.* When the subject is a plurality of individuals taken collectively. A collective noun is, for Logical purposes, Singular: as, "This crowd is tumultuous," "An army consists of soldiers."

Singular.

9. Singular Judgments, for all Logical purposes, may be accounted as Universals, since in them, the whole subject is spoken of, and they are subject to the laws of Universals.

Singular Judgments equivalent to Universals.

In like manner, when any Definite part of the Subject is taken, it may be considered as a universal. For the whole class denoted by the subject-name with its limiting adjuncts is spoken of—Thus "these men are natives of Ireland."

Also a Definite part of the Subject.

It is in place here to add, that Judgments are further distinguished as Simple and Compound. Judgments are Simple when, in fact as well as form, there is but one subject and one Predicate, as, "men are rational animals." A Judgment is Compound when, though simple in form, by a plurality of subjects or predicates, there is in force and effect a plurality of Judgments. Thus, "Peter, James, and Thomas were Apostles," amounts to three propositions, one affirming of Peter, another of James, and another of John, that he was an Apostle. "Men are rational, accountable and immortal," may be divided into three propositions, each having "men" for the subject, but one having the predicate "rational," the other "accountable," etc.

Simple and Compound Judgments.

Sect. IV.—Quality of Judgments.

10. The differential Quality of a Judgment is that it affirms or denies the agreement of Subject and Predicate. Hence in respect of Quality, Judgments are either Affirmative or Negative. ☞ Let the learner remember that the *Logical* Quality of a Judgment refers to its being Affirmative or Negative. The truth or falsity of a Judgment is of course of supreme importance.

Quality respects Affirmation or Negation.

But this pertains to its matter, not to its form, with which alone formal Logic concerns itself.

11. A Proposition is Affirmative or Negative, according as it has not or has, a negative Copula; *i. e.* when, whatever be the form of expression, the real force of a negative does not or does, fall on the Copula. Thus, "no iron is silver," is negative, for it asserts of all iron that it is not silver. "A person not vicious is virtuous," is affirmative, because the force of the negative does not fall on the Copula but on one of the terms. "A few men are wise," is affirmative; "but few men are wise," is in reality negative, for it is equivalent to "most men are not wise."

Quality in the Copula.

Examples.

Judgments then as to Quantity and Quality, as thus unfolded by the old Logicians, are four, which they have been accustomed to mark by the Symbols, A. E I O., as follows:

The four Logical Judgments and their Symbols

Universal Affirmative, ... A.
Universal Negative, ... E.
Particular Affirmative, ... I.
Particular Negative, ... O.*

* The additional Judgments recognized by recent Logicians will be noticed in due time.

Sect. V.—Distribution of Terms in Judgments.

12. Of the foregoing Judgments all Universals and no Particulars distribute the Subject.

Universals distribute the Subject. Negatives the Predicate.

All Negatives and no Affirmatives distribute the Predicate.

The reason of the first rule is obvious, for in Universals the whole subject is spoken of Distributively.* In Particulars only a part of it.

No Negative Judgment can hold good unless it cuts off the whole of the Predicate from the subject. Thus, if we say, "some men are not poets," the whole of the class of poets is cut off from these "some men." "No men are perfect," cuts off the whole of the class "perfect" from the class men.

Sect. VI.—Relation of Judgments.

13. The Relation of Judgments has respect to the manner of the connection between the subject and Predicate. In this respect Judgments are either Categorical or Hypothetical.

Relation either Categorical or Hypothetical.

* Collective Nouns are no real Exception, since in a Logical sense, they are individuals and form the subjects of Singular Judgments.

14. A Categorical Judgment asserts or denies the agreement between the subject and Predicate, simply and unconditionally, as, "Brutus killed Cæsar," "a traitor is not a patriot."

Categorical.

15. A Hypothetical Judgment asserts or denies such agreement upon a condition, viz: of the truth or falsity of some other Judgment. Thus, "if crops are large, food is cheap." "This man is either holy or unholy."

Hypothetical.

16. Hypothetical Judgments are of three kinds: Conditional, Disjunctive, and Dilemmatic.

Three kinds of Hypothetical Judgments.

17. The Conditional Judgment affirms such a relation between two others, respectively called Antecedent and Consequent, that, if the former be true, the latter is true also, as, "if the sun shines, it will give heat." Conditionals are indicated by the particles, "if," or its equivalents, "when," "in case of," etc.

Conditional Judgments.

Antecedent and Consequent.

18. The conditional, like all hypotheticals, has in it a categorical element, *i. e.* it asserts categorically a certain relation between the Antecedent and Consequent; such, that, if the former is true, the latter is true; and if

Hypotheticals have a Categorical Element.

the latter is false the former is false. It often expresses the relation of cause and effect. If the cause operates the effect will follow. It is to be observed that a conditional does not assert the truth of either of its members, but of the relation between them. It may assert, not only a causal relation, but the truth of a certain argument. Thus, "if drunkards drink what intoxicates, A. B. drinks what intoxicates." This is not an assertion either that drunkards, or A. B. drink what intoxicates; nor that the former is the cause of the latter; but that there is such a relation between the two, that if the former be true the latter is true. A certain fact, however, is by implication asserted as the foundation of this relation, viz., that A. B. is a drunkard.

Causal Relation often in Conditionals.

Do not assert the truth of either member.

Other relations.

19. Disjunctive Judgments assert the connection between the predicate and the subject, with an alternative indicated by the particles, either and or. Thus, "it is either Spring, Summer, Autumn, or Winter." The force of it is that if one member be affirmed, all the others are denied. If one is denied, then some one of the residue is true. This is founded on the law

Disjunctives assert with an alternative.

of Excluded Middle. A judgment or its contradictory must be true, and there is no middle between them. So conditionals are founded on the law of Sufficient Reason. Of categoricals the affirmatives are founded on the principle of Identity, and the negatives on the law of Contradiction.

Founded on Excluded Middle.

Its members mutually exclusive.

20. Hence, in order to any valid conclusion from the affirmation or denial of either member of a disjunction, these members must be mutually exclusive. Indeed such alone are genuine disjunctives. Disjunctives must not be confounded with Partitive Judgments, which, under the form of a disjunctive, simply predicate of a genus its several species; as, "all Africans are either bond or free." This is but dividing the genus into its component parts or species. It differs from the disjunctive in this, that the predicates are affirmed concurrently, and not alternatively, of the subject. The affirmation of the one is not, as in a pure disjunctive, a denial of the other, although the predicates are still mutually exclusive with regard to the portions of the subject to which they respectively belong.

Differ from Partitives.

21. Dilemmatic Judgments involve a combination of the conditional and disjunctive. Thus, "if A. B. succeeds, he will either rule or ruin." Here the disjunction is in the consequent of the conditional. It may also be in the antecedent. "If man is either good or ill deserving, he is a moral agent." Dilemmatic Judgments.

Sect. VII.—Substitutive Judgments.

22. Substitutive Judgments are those which being affirmative have a distributed predicate. This distribution of the predicate cannot be known from the mere form of expression. As we have already seen, affirmatives as such, do not distribute the predicate. To say that men are mortals, is merely saying that they are in the class of mortals. They in fact comprise a part but not the whole of mortals. But if we say, "men are rational animals," we mean all rational animals, for there are none but men. This, however, does not appear from the affirmative form of expression, any more than, if we were to say, "men are animals." We know it from other evidence. "Rational animals" is the definition of Substitutives defined. How known.

men, and is, therefore, co-extensive with it. In all cases of Definition then, and in all the kinds of Definition which have been pointed out, we have Substitutive Judgments.

All Definition Substitutive.

23. Judgments of this kind are called Substitutive, because the predicate may be substituted for the subject without limiting the quantity, either of the judgment, or of the predicate substituted. If we define men to be rational animals, we can say that "all rational animals are men." If we say that "Maine, New Hampshire, etc., are the New England States," we can, by simple substitution, say that "the New England States are Maine, New Hampshire, etc."

Why called Substitutive.

24. Substitutive Judgments are either Particular or Universal. Of these latter we have already given examples. The former are such as, "some stars are planets," *i. e.* all the planets: "some men are poets," *i. e.* all poets.

Either Particular or Universal.

25. Affirmative Judgments, in which the predicate is undistributed, are called Attributive, because they affirm an attribute of the subject, without taking this attribute in its whole extent, or substitutively. Thus, "men are rational."

Attributive Judgments.

26. The importance of Substitutive Judgments will appear, when we come to treat of the subject of reasoning. They render many processes of reasoning valid, which would otherwise be invalid, owing to the non-distribution of affirmative predicates, as will be explained in the proper place.

Importance of Substitutive Judgments.

27. The reason why logicians who have recognized this class of judgments, have treated this subject as belonging to the Relation of judgments, or as concerned with a peculiar class of them, in respect to the manner of the connection of the subject and predicate, is, that it exhibits the quantity of the predicate as related to the subject. Indeed every affirmative judgment, when fully explicated in language, becomes an equation of the subject and predicate as to quantity, and so a Substitutive Judgment. This will appear if we explicitly quantify the predicate, *i. e.* fully express in words what we mean in thought. Thus, if we say, "all men are mortals," we mean, "they are (*i. e.* =) some mortals." "All men are rational animals," means "all men are (*i. e.* =) all rational animals."

Why this pertains to the relation of Judgments.

Equation of Subject and Predicate.

28. Substitutive Judgments are indicated respectively, the universals by the letter U, and the particulars by the letter Y. Thus, we have six different kinds of judgments, designated by their several symbols as follows:

The Symbols of Substitutive Judgments.

Universal Attributive,	A.
Particular Attributive,	I.
Universal Negative,	E.
Particular Negative,	O.
Universal Substitutive,	U.
Particular Substitutive,	Y.

29. [Besides these, Sir William Hamilton has undertaken to develop two others; viz., Universal and Particular Negative Judgments with undistributed predicates, which he marks by the respective symbols η and ω. But undistributed negative predicates are so contrary to all normal thought and language, that, at best, they are useless, and need not claim our attention. The Judgments, "No men are some animals," "and some men are not some animals," are awkward, insignificant,

Negatives with undistributed Predicates.

Insignificant and worthless.

and worthless, being nearly, if not quite incapable of real contradiction.*]

Sect. VIII.—Analytic and Synthetic Judgments.

30. Analytic Judgments are those in which the predicate is involved in the very Conception or Definition of the subject. As, "man is rational." "Quadrupeds are four-footed." Analytic Judgments.

* They cannot, with slight exception, be opposed by contrary or contradictory propositions, in any normal use of language.

The following table in which A stands for a distributed, and I for an undistributed term, and the letters f and n respectively for an affirmative or negative copula, exhibits at a glance the import and force of the Eight Judgments recognized by Hamilton.

A.	Afi.	All are some.	All men are mortals.
E.	Ana.	Not any is any.	No men are angels.
I.	Ifi.	Some are some.	Some trees are beautiful.
O.	Ina.	Some are not any.	Some coins are not silver.
U.	Afa.	All are all.	All men are all rational animals.
Y.	Ifa.	Some are all.	Some men are all the poets.
η.	Ani.	Not any are some.	No planets are some stars.
ω.	Ini.	Some are not some.	Some trees are not some oaks.

ω is without force because not contradictory to nor inconsistent with any other proposition. η may indeed have greater force. But this is seldom important in actual thought. Both judgments indeed are rather conceivable than actual in normal thought, and for practical purposes, without assertory force.

They therefore require no proof. They are evident simply from the analysis of the subject. Hence they are *a priori*, *i. e.* known from the conditions given, if not always in the most absolute meaning of *a priori*, yet from the definition of the subject.

31. Synthetic Judgments are those in which the predicate adds to the conception or definition of the subject. They, therefore, require proof. Thus: "laurel-water is poisonous," "horned animals are ruminant," "the conception of a perfect being involves his existence." Synthetic Judgments are, with a qualification to be noted, *a posteriori*. The Formal Sciences, and those which deal with necessary truth, furnish us a peculiar class of Judgments that are both synthetic and *a priori*. All the demonstrated propositions in Geometry, *e. g.* are *a priori*. Yet they are not a part of the definition. They are not immediately suggested or implied by it. They require to be proved by a chain of reasoning from the definitions, more or less extended. Yet this reasoning is *a priori*. The same is true of most of the principles of Logic. In this sense we have Synthetic Judgments *a priori*. They are, in truth

Synthetic Judgments.

A posteriori. Exception in Formal Sciences.

How they give Synthetic Judgments a priori.

partly analytic, in that they are ultimately evolved from the definitions; synthetic in that they require proof beyond the mere statement of the definition. The origin of this use of the terms analytic (*αναλυω*, to take asunder), and synthetic (*συντίθημι*, to put together), is evident from their etymology.

The terms Explicative and Ampliative have, for obvious reasons, been employed to denote the same properties of Judgments as Analytic and Synthetic.

Explicative and Ampliative.

Sect. IX.—The Modality of Judgments.

32. The Modality of Judgments respects the possibility, certainty, or necessity of the connection of the predicate with the subject. This, however, really belongs to the *meaning* of the predicate rather than to the copula, or any part of the logical form of the judgment. Strictly, therefore, it pertains to the matter rather than the form of the judgment, to Metaphysics instead of Logic. It belongs, accordingly, rather to applied than to pure Logic. To this we shall therefore defer it, although it is sometimes treated at this point.

The Modality of Judgments belongs to Applied Logic.

Sect. X.—Plurative Judgments

33. Plurative Judgments are those in which more than half, but not all of the subject is taken; as, "Most men are vain." Of a similar nature are Numerically Definite Judgments, *i. e.* those in which a definite number or numerical proportion of the subject is taken.

Plurative Judgments defined.

Numerically Definite Judgments.

Both of the foregoing have some importance as giving rise to a peculiar kind of valid syllogism which will be explained in its proper place. See chap. V., sect. I. 5.

Sect. XI.—Conversion of Hypotheticals into Categoricals.

34. It has already been shown that in every Hypothetical Judgment there is a categorical element, which affirms or denies the given hypothetical relation between certain categorical judgments. This being so, by a slight change of phrase, they may be made Categorical in form. This can be done, as follows,

Hypotheticals have a Categorical element.

a. Conditionals may be so converted by substituting for the particles "if," "when," etc., which have a conditional force,

Conditionals how turned into Categoricals.

such phrases as "the case of," the "circumstances in which," etc. Thus the conditional, if A is B, X is Y, is the equivalent of, "the case of A being B is the case of X being Y," which is a categorical. The conditional, "if the thermometer is at zero, ice forms rapidly," may be transformed into, "the case of the thermometer being at zero," or "the case," or "the circumstance," or "the time in which the thermometer is at zero, is that in which ice forms rapidly."

Abbreviations in the case of only three Terms.

Certain Abbreviations are practicable when the same terms are found in both antecedent and consequent. Thus the conditional, "if Peter is a drunkard, he (Peter) is degraded," is equivalent to "every drunkard is degraded," otherwise it could not be true.

Disjunctives. By Colligation.

b. Disjunctives may be turned into Categoricals by using all their members for one of the terms, and the phrase, "possible cases," or the like, for the other, thus forming a judgment by Colligation, which, as we have seen, is the opposite of Logical Division. Thus: "This season is either Spring,

Summer, Autumn, or Winter," is equivalent to either of the categoricals, "the possible cases in regard to this season," or "the only alternatives in regard to it, are Spring, Summer, Autumn, Winter."

By first being turned into Conditional.

As has been shown before also, Disjunctives may be turned into Conditionals, by taking the contradictory of one of their members for the antecedent, to which the other members become consequents. Thus, in the foregoing example, "if it is not Spring, it is either Summer," etc. When once a conditional, it can be made a categorical, according to the rules already given, *e. g.* "The case of its not being Spring, is the case," etc.

Dilemmatic Judgments to be Resolved.

c. Dilemmatic Judgments being compounded of Conditionals and Disjunctives, may be resolved into these, and each of these may be changed into categoricals, according to the methods just indicated. Thus, the Dilemmatic Judgment, "If Æschines did or did not join in the public rejoicings, he was either inconsistent or unpatriotic," may be analyzed; "If he joined, etc., he was inconsistent;" "If he did

not join, etc., he was unpatriotic;" "But he did or did not join;" "He was either inconsistent or unpatriotic." These may be turned into Categoricals by the methods already prescribed.

CHAPTER IV.

REASONING—IMMEDIATE INFERENCE.

SECTION I.—INTRODUCTORY REMARKS.

1. THE next stage of Thought after the formation of Judgments, is that of deriving from judgments given other judgments founded upon them. This is Reasoning.

Reasoning Defined.

2. Reasoning is by inference from one Judgment to another derived from it; or from two judgments to a third, which could not be derived from either alone, but flows from both combined. The former is called Reasoning by Immediate Inference, the latter by Mediate Inference, *i. e.* from one judgment through the medium of another; or more strictly, as it will more fully appear, through a middle term, *medius terminus*, common to both the judgments given, by means of a common or opposite relation to which, the two

Mediate and Immediate Inference.

terms of the conclusion are found to agree or disagree with each other. These two kinds of reasoning will be severally treated in their order.

Three kinds of Immediate Inference.

Opposition, Conversion, Equipollence or Infinitation.

3. Immediate Inference, *i. e.* inference of one judgment from another, is of three kinds, termed Opposition, Conversion, and Equipollence or Infinitation. And first of,

Sect. II.—Opposition.

Opposition defined.

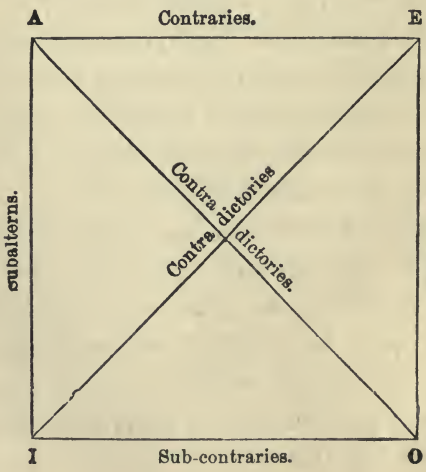
4. Opposition exists between judgments having the same subject and predicate, but differing in quantity, or quality, or both. Thus, "all A is B," and "some A is not B," are opposed. They differ both in quantity and quality. This is the strongest kind of opposition, called contradictory. From any judgment whatever, an inference can be made regarding its contradictory, or which is the same thing, any affirmation or denial regarding either of two contradictories, warrants an inference in regard to the other. Thus, if we take the two contradictories, "all men are mortal," "some men are not mortal," when either is true the other is false; when either is false the other is true.

Contradictories.

5. Besides the Contradictories, we have the Contraries A and E, and the Sub-Contraries I and O, which respectively differ in quality alone. Also the Subalterns A and I, E and O, in which the members of each respective pair differ from each other only in quantity. In each pair of these the Universal is called the Subalternans, the Particular the Subalternate. All these forms of opposition are brought compactly and clearly to view by the following ingenious and simple diagram, which has been devised by logicians.

Contraries, Sub-Contraries, Subalterns.

Subalternans. Subalternate.



6. The Laws of Inference in the case of judgments in opposition are, in brief, as follows: Laws of Inference in Opposition.

a. Of Contradictories one or the other must be true, both cannot be true. And by the law of Excluded Middle no intermediate between them can be true. Therefore, from the truth of either of two contradictories, it follows that its opposite is false; and from the falsity of either, the truth of the opposite may be inferred. Re-Contradictories.

b. From the truth of either Subalternans, the truth of its Subalternate follows. From its falsity nothing follows with regard to the Subalternate; from the truth of either Subalternate nothing follows in regard to its Subalternans. From the falsity of the Subalternate the falsity of the Subalternans results. Subalterns.

c. From the truth of a Contrary the falsity of the opposite Contrary follows. But from the falsity of one Contrary, nothing follows in regard to the other. Contraries.

d. From the truth of either Sub-Contrary nothing follows in regard to the other Sub-Contrary. But from the negative of one of them, it follows that the other must be true. Sub-Contraries.

7. From this it appears that the Opposition between Contradictories is far the most important and fruitful of inferences. The most available mode of proving the truth of many propositions, is to prove their Contradictories false.

Contradictory Opposition most important.

8. The foregoing view exhausts opposition as between the four fundamental judgments A E I and O above recognized by the old logicians. But if we bring in the additional substitutive judgments U and Y already considered, they lay a foundation for other forms of Opposition.

Other forms of Opposition created by the Judgments U and Y.

a. For other forms of Contrary Opposition. The characteristic of this kind of opposition is, that of two judgments opposite in quality whatever their quantity, both judgments may be false, but cannot be true together. Thus A and E may both be false, but cannot both be true together. But the same is true of E and U. Thus, it is false alike that "no men are poets," and that "all men are all the poets." It is true that "all men are all rational animals," false that "no men are rational animals."

Other forms of Contrary Opposition.

b. Out of the Opposition of these Substitutive

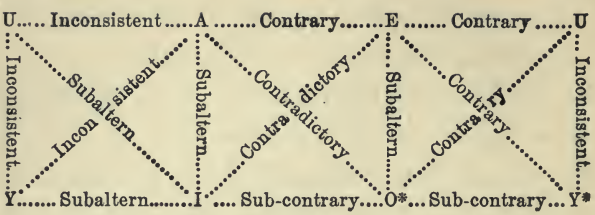
Judgments to each other, and to other judgments arises what has been named Inconsistent Opposition. This obtains between judgments opposed in quantity but not in quality, which cannot both be true, though both may be false, at the same time. Thus the opposed judgments A and U cannot both be true of the same subject and predicate, unless A be considered, as it was by old logicians, to include U. It cannot be true that all men are all the animals (U), and that all men are only some animals (A). But A and U may both be false, as in any subject and predicate in which negatives or particulars only are true.

Inconsistent Opposition.

c. Subaltern Opposition exists when there is more distribution in either term of one judgment (the Subalternans) than in the corresponding term of the other (the Subalternate). Accordingly, this kind of opposition exists between U and I, also between Y and I, for from positing either U or Y, I may be inferred. But from I neither U nor Y can be inferred.

New Subaltern Opposition.

9. Applying these principles, and extending the diagram of Opposition already given to include U and Y, we have the following result:



10. Opposite Judgments must have the same subject and predicate, not only in sound but in sense. "Bread is heavy," and "bread is not heavy," are not opposed, if in the former case "heavy" be used to denote imperfect fermentation, in the latter to denote specific gravity as compared with lead.

* Some writers, among whom is Thomson in his *Laws of Thought*, class the opposition between A and O, as Contrary instead of Contradictory, and admit only E and I to be Contradictories. He says, "We cannot tell from the removal of O whether we ought to replace it by A or U." This, however, though theoretically true, hardly calls for a deviation from the established use of terms in practice. Would not this argument abolish the contradiction between E and I? If E be removed, we do not, from that fact, know whether it may not be replaced by A, U, or Y, as well as I. We only know that as much as I is true. In like manner we know that certainly as much as A, and possibly as much as U, is true if O be removed. This gives them both the power of contradictories. How much more is true in any case must be learned from other sources. Y is a kind of false subcontrary to O. If it be true, O is true.

Sect. III.—Conversion.

11. A second mode of immediate inference is by Conversion. Propositions or judgments are converted when the predicate and the subject change places, in such a way that the converse is an inference from the convertend or judgment converted. Conversion Defined.

12. In order that Conversion may be illative, or give rise to a legitimate inference, no term must be distributed in the converse which was not distributed in the convertend: otherwise more would be spoken of in the conclusion than in the premise. This was the rule of the older logicians. Law of Distribution of Terms.

13. Hamilton and his school, however, maintain, not only that no term should be distributed in the converse which was undistributed in the convertend, but that all terms distributed in the latter should be distributed in the former. Hamilton's Law.

14. Conversion, in order to be logical, according to these principles, sometimes requires a change in the Quality or Quantity of the convertend. Hence result the following modes of Conversion. Change of Quantity or Quality sometimes necessary.

A. Simple Conversion is when there is no change in either Quality or Quantity.

Simple Conversion.

B. Conversion by Limitation, sometimes called *per accidens*, is when the quantity is changed from universal to particular.

By Limitation or per accidens.

C. When the quality is changed, it is said to be by Negation or *Contra-position*.

Negation and Contra-position.

15. Accordingly,

a. A, which distributes the subject but not the predicate, must be converted by Limitation from universal to particular, and therefore, according to the old Logic, which does not recognize the distribution of affirmative predicates, becomes I; but with a quantified and distributed predicate it becomes Y. Thus, "all men are mortal," becomes in the former method, "some mortals are men," which is I; and in the latter method, "some mortals are all men," which is Y. It is justly argued that the latter is the only perfect Conversion, because it alone enables us, by reconversion to regain the original convertend. This ought to be possible in thorough conversion. From

How to Convert A.

By old Logic A. becomes I.

In perfect Conversion A. becomes Y.

"some mortals are men," *i. e.* "some men," we can only get by re-conversion, "some men are mortals." But from "some mortals are all men," we readily get back the original convertend, "all men are mortals." Of course from Y comes I, its subalternate.

Y involves I.

b. E., which distributes both terms, may be converted simply, and remains E after conversion. If, "no men are angels," then "no angels are men."

E Converted simply.

c. In like manner I, which distributes neither term, may be converted simply. If some Americans are Indians, then some Indians are Americans.

Also I.

d. O distributes the predicate but not the subject. Consequently, if it were converted without changing its quality, the subject undistributed in the convertend, would be distributed in the converse by being the predicate of a negative. Thus, "some quadrupeds are not horses," would become "some horses are not quadrupeds," which is obviously illogical as well as false.

O converted by contraposition and why.

In order to avoid this, the negative particle is transferred from the copula to the predicate, so that

the convertend becomes I, which may be simply converted. Thus, for "some quadrupeds are not horses," say, "some quadrupeds are [not horses]$^{\text{Pred.}}$," or "things not horses." This is I, which converted simply becomes, "some things not horses are quadrupeds." Here conversion is by contraposition.

Conversion of A and U by contraposition.

16. A and U, *i. e.* all affirmatives which have the subject distributed admit of this sort of conversion. Thus,

A. "All men are rational," may be converted into
E. "Whatever is not rational is not a man."
U. "All men are rational animals," may become
E. "Whatever is not a rational animal is not a man."

U converted simply and thence into I.

17. U may be converted simply. Thus, "All men are rational animals." Therefore, "All rational animals are men." This is U, and by subalternation will give I also.

Y into A and I.

Y may be converted into A, which by subalternation yields I. Thus, "Some men are poets" (*i. e.* all the poets), yields A. All poets are men.

Summation.

18. The several kinds of judgments therefore may be converted as follows:

A may be converted into Y and thence I.

E into E, and thence O.

I into I.

O into I indirectly by contraposition.

U into U, and thence I.

Y into A, and thence I.

A and U may also be converted by contraposition.

Sect. IV.—Other Modes of Immediate Inference.

19. Besides Opposition and Conversion, the standard modes of Immediate Inference formerly recognized by logicians, several other forms of it deserve mention.

Other forms of Immediate Inference.

A. By Reciprocal Change of Positive and Privative Conceptions.

20. If we take any pair of Positive and Privative, or as they are styled by some, Infinitated Conceptions, as has before been shown, they comprise, taken absolutely, all being, or the universe: and taken most narrowly, they include all the members of the genus which is the particular object of thought. Thus "virtuous" and "not virtuous,"

Change of Positive and Privative Conceptions.

taken absolutely, include the universe of actual and possible being. But practically, they are only used in reference to beings capable of virtue, *i. e.* moral beings, and, in ordinary cases, are applied to none but mankind. Supposing the latter to be spoken of, all are included in the virtuous and non-virtuous, and whatever men are not one are the other.

To affirm the Positive is to deny the Privative and vice versa.

Therefore, to affirm a Positive Conception of any subject, is the same as to deny its corresponding Privative, and *vice versa*. It is often convenient in such cases, instead of an awkward and confusing use of the particle "not," in order to mark the Privative contradictory, to use the particles *in* or *un* to form a single compound privative word—as *in*consistent for *not* consistent, *un*wise for *not* wise—or to use any word of corresponding privative import, without any explicit negative particle, as *foolish* for *unwise*, *soft* for *not hard*.

Rules for such Conversion.

21. This sort of immediate inference is governed by the two following rules.

a. If the predicate be changed from Positive to Privative, or the reverse, change the quality of the judgment. Thus, "all men are rational," "no men

are (not rational), *i. e.* irrational," and by conversion, "no irrational beings are men."

b. To change the subject in like manner, first convert the proposition: thence change the subject (now become predicate), from positive to privative or the reverse, and change the quality of the judgment. Or, (what is the same), convert the judgment, and proceed as in rule first. Thus,

"Some men are (all the) poets." By conversion,
"Some (or all) poets are men."
"Some (or all) poets are not beings who are not men."
"No trees are stones." By conversion,
"No stones are trees."
"All stones are things not trees."

These methods of immediate inference may be applied to all the varieties of propositions.

B. IMMEDIATE INFERENCE FROM DISJUNCTIVES OR PARTITIVES.

22. In a Disjunctive or Partitive Judgment, it is immediately evident that whatever of the objects included in it belongs to one of its members, is not included in any of the others, and whatever is not included in it, does belong to one of the others. Thus, "The

From Disjunctives and Partitives.

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seasons are either Spring, Summer, Autumn, or Winter." "Spring is neither Summer, Autumn, nor Winter," and "whatever seasons are not Spring, are either Summer, Autumn, or Winter."

C. From a Combination of Predicates.

23. If it be known of man that he is rational, also that he is animal, also that he laughs, then these Predicates may be united in one judgment, which may be A or U, according to circumstances—in the present case U—Thus: "Man is a rational animal that laughs."

By uniting Predicates.

Other Formulæ furnish materials for immediate inference too numerous and obvious to require minute specification. "Howard was a philanthropist," therefore philanthropy has a real existence. "The President is the supreme executive," therefore to assail the President is to assail the supreme executive.

Other Formulæ.

24. Some have maintained that these processes of Immediate Inference are unimportant, because the conclusion contains nothing not previously contained in the premise. But if this objection be valid it lies against all reasoning. It is further objected that the conclusion is identical with the premise. This

The importance of Immediate Inference shown.

is an error. It will hardly be claimed in regard to inference by opposition, especially contradictory opposition. In conversion the subject or principal notion on the judgment is changed. Equivalent changes from the premise to the conclusion occur in other forms of immediate inference. Few persons who have not made Logic a study, can state with accuracy the exact illative converse of any, especially of all the fundamental logical judgments, as they understand, who have had experience in teaching Logic. An eminent logician says, "Could any person not accustomed to exercises of this kind, draw out fully *all* his own meaning, when he utters the simplest proposition? The judgment 'all men are mortal' (a plainer cannot be found), tells us that man is one species in the class of mortal beings—that the mark of mortality should always accompany our notion of man—that the word mortal is a name which may rightly be given to man—that, if *all* are mortal, any one man is—that any statement which affirms that no men are mortal, must be quite false—that even the statement that *some* men are not mortal is equally false—that since man is contained in the

Conclusion not Identical with the Premise.

Explication of "All men are mortal," into other judgments.

class of mortal beings, which is a wider class, it would be wrong to say all mortal things are men—that, however, the assertion "some mortals are men," would be true enough—even "some mortals are *all* men"—that no men can be immortal—that any immortal beings must be other than men—that mortality really exists, being found in man, whom we know to exist—that a man with immortal hopes is a mortal with immortal hopes—that (since heaven is immortality) a man expecting heaven is a mortal looking for immortality—that he who honors a man, honors a mortal. Thus from this simple judgment fourteen judgments have unfolded themselves, or, as some would say, the judgment has been put in fifteen different ways, in the last three of which only is any new matter introduced. And yet any man of common sense would say that his proposition really implied them."—*Thomson's Laws of Thought*, pp. 191–2.

CHAPTER V.

REASONING—MEDIATE INFERENCE.

Section I.—Introductory Remarks.

Immediate Inference, as we have seen, is of one Judgment from another without the intervention of any third judgment or third term.

1. Mediate Inference is from two judgments given as premises to a third founded upon them, in which the two terms of the conclusion are found to agree or disagree with each other, through a third or middle term with which they have each been compared in the premises. Thus, all M is P, all S is M, ∴ All S is P. Here all S is declared to be P, because it has previously been affirmed to be M, and all M to be P. Or if we take a negative conclusion, "No stones are trees, This oak is a tree, ∴ It is not a stone." Here, oak, in the conclusion, is declared not to be a stone, because it is a tree, and no trees are stones.

Mediate Inference is from two Judgments given to a third.

Through a Middle Term.

2. It is obvious that the ultimate ground of Mediate Inference, as shown in the above examples (and the same may be shown of all others), is reducible to the principles of Identity and Contradiction or rather Non-contradiction. In the first case S and P agree—are one with each other,—because they each agree with, are the same as, M. Oak and stone do not agree with each other, because the one is, the other is not, a tree. To say that they are one, would therefore be a contradiction.

Founded on Identity and Contradiction.

Illustration.

3. This process of reasoning from two judgments given, to a third derived from them, through a middle term, is called an Argument, (from *Argumentum*, proof) and, when stated in regular logical form, so that the connection of the premises with the conclusion is immediately evident, it is called a Syllogism, *συλλογίσμος*, *i. e.* collecting the elements given in the premises into a conclusion.

Argument.

Syllogism.

4. The subject of investigation now before us, therefore, is the doctrine of Syllogisms.

A Syllogism, like all other reasonings, consists of two parts, that which is to be proved, and that by which it is to be proved

Parts of the Syllogism.

Of these, in whatever order they may stand, the latter are called the Premises. These Premises are Major and Minor.

Premises.

The Major Premise is that in which the Major Term is compared with the Middle, whatever may be the order in which they stand.

Major Premise.

The Minor Premise is that in which the Minor Term is compared with the Middle.

Minor Premise.

The Premises, as the word implies, are put before the Conclusion, when the syllogism is arranged in regular logical order. Thus:

Order of Premises and Conclusion.

"All conquerors are tyrants.
Buonaparte was a conqueror.
He was a tyrant."

In this case the Conclusion is connected with the Premises by some inferential particle, such as "therefore," "hence," etc.

But it is more common, and quite as natural, to adopt the reverse order in actual reasoning—to put the Conclusion first and the Premises afterward. Thus: "Buonaparte was a tyrant for he was a conqueror, and all conquerors are tyrants." And fre-

quently, in either case, not more than one premise is expressed, the other being understood and obvious. Thus: "Many voters are tools of demagogues because they are ignorant." "Free government will continue since the people are virtuous." This, regularly drawn out would be,

"A virtuous people will preserve a free government.
This people is virtuous.
∴ It will preserve a free government."

Synthetic and Analytic Syllogisms.

A Syllogism in which the premises are stated first is called Synthetic, because it puts together the premises in order to form the conclusion.

When the conclusion is stated first, it is called Analytic, because this conclusion is analyzed into the proofs out of which it grows.

Major Term.

The Major Term is the predicate of the Conclusion.

Minor Term.

The Minor Term is the subject of the Conclusion.

Has three Judgments.

Hence every Syllogism must have three, and but three, Judgments. The Major Premise, the Minor Premise, and the Conclusion in which the major and minor terms are compared with each other.

Every Syllogism must have three, and but three, Terms; the Major, Minor, and Middle. If there be four Terms, either in form or in fact (from the ambiguity of either of them), the two terms of the conclusion will not have been compared with one Middle Term, and no conclusion can follow.

Three Terms.

5. From the principles of Identity and Contradiction, the following Canons for testing the validity of all Syllogisms result.

Canons of the Syllogism.

a. If the Major and Minor Terms, each being compared with the same third or Middle term, both agree with it, they agree with each other. This underlies all Affirmative Conclusions.

Canon of Affirmative Conclusions.

b. If of the Major and Minor Terms, both being compared with the same third term, one agrees and the other disagrees with it, they disagree with each other. This is the foundation of Negative Conclusions. Therefore if one premise be negative, the conclusion must be negative.

Of Negative Conclusions.

c. If they both disagree with the same third term, no conclusion follows as to whether they agree or disagree with each other. This is the case of Negative Premises,

Negative Premises give no Conclusion.

from which there can be no Conclusion. Thus, from "A bird is not a sheep," "a robin is not a sheep"—nothing can be inferred.

d. The Middle Term must be distributed at least once in the premises, otherwise the Minor Term may be compared with one part and the Major with another part of it. From,

Middle Term must be Distributed.

> "Some men are poets,
> Some men are Indians,"

Nothing follows.

Plurative Judgments, however, give rise to a peculiar class of valid Syllogisms with an undistributed middle. Thus:

> "Most men have some kind of religion,
> Most men are uncivilized,
> ∴ Some uncivilized persons have some kind of religion."

The same is true of numerically definite Judgments. Thus:

> "60 out of this 100 are unreflecting,
> 60 out of this 100 are restless,
> ∴ 20 restless persons are unreflecting."

e. No term may be distributed in the conclusion

which was not distributed in the premises. This, which is Illicit Process, is furtively speaking of more in the conclusion than was contained in the premises. Thus:

No Illicit Process.

> "All beasts are animals,
> Birds are not beasts,
> They are not animals."

f. From Particular Premises, of which Y is not one,* nothing can be inferred. With none but the particular judgments I and O of the old logicians in the premises, no conclusion can follow, because, if both were I, no term would be distributed, whence would result an undistributed middle. From "some men are heroes," and "some men are poets," nothing can be inferred. If both premises be O, they are both negative, and no conclusion can follow. If one be O and the other I, the middle term must be the predicate of O in order to be distributed, and in that case all the other terms will remain undistri-

Particular premises give us no Conclusion.

* The following is a valid conclusion from the particular judgments Y and I.

> Y. Some trees are all the oaks.
> I. Some oaks are white oaks.
> ∴ Some white oaks are trees.

buted. But, one premise being negative, the conclusion must be so likewise. This would distribute the major term in the conclusion, which by supposition was undistributed in the premises. Illicit process results. Thus:

"Some men are not cultivated,
Some poets are cultivated,
Some poets are not men."

g. If either Premise be particular, the Conclusion must be Particular. In other words, a universal conclusion requires both premises to be universal.

Conclusion particular when either premise is so.

If the universal conclusion be A, then the subject of it must be distributed in the premises, and must therefore be the subject of one of them, since being both affirmative, neither can distribute the predicate. For the same reason the middle term will be undistributed in that premise, being then the predicate of an affirmative. Therefore the middle term must be the subject of the other premise, which must also be universal, in order that it may be distributed. Thus a universal affirmative conclusion requires both premises to be universal.

If the universal conclusion be E, then both its terms must be distributed in addition to the middle

term in the premises. This requires both premises to be universal and one of them negative, or both negative and one universal. The latter is impossible as no conclusion can come from two negative premises. Therefore the premises must be both universal.

The principle that one negative or one particular premise renders the conclusion respectively negative or particular, logicians have expressed by saying that the conclusion follows the weaker part. The whole of these canons have been condensed into the following Latin lines:

> "Distribuas medium nec quartus terminus adsit,
> Utraque nec præmissa negans, nec particularis:
> Sectetur partem conclusio deteriorem,
> Et non distribuat nisi cum præmissa, negetve."

This reasoning, however, applies only to syllogisms in the old Logical Judgments, A E I and O. Syllogisms with U or Y in the premises, may have universal conclusions with one premise particular. Thus:

U. "All men are rational animals,
Y. Some men are all the poets,
∴ All the poets are rational animals."

A. "All men are rational,
Y. Some men are all the Polynesians,
∴ All the Polynesians are rational"

U. "Animals are all bodies having sensation,
Y. Some animals are all oysters,
∴ All oysters have sensation."

Sect. II.—Moods.

(For Moods as affected by Substitutive Judgments, see Appendix B.)

Mood Defined.

6. The Mood of a Syllogism is the relation of its several judgments to each other, with reference to their respective quantity and quality, these being designated by the symbolic letters A E I O. The Mood of a syllogism, whose premises and conclusions are universal affirmatives thus becomes A A A. If the major premise were universal affirmative, the minor universal negative, and the conclusion universal negative, it would be A E E, etc., etc.

Number of Moods.

The possible combinations of these four kinds of propositions are of course $4 \times 4 \times 4 = 64$. But most of these are invalid as involving violations of some of the preceding canons. Thus E E E, E O O, and others, are bad on account of negative premises. I O O and others, for particular premises. I E O, for illicit process. Sifting out all moods that are thus invalid, only eleven valid ones remain. And of these only a part are valid in any one figure.

Only eleven valid Moods.

Sect. III.—Figure.

7. The Figure of a Syllogism depends upon the situation of the Middle Term in the premises.

The Figures as fixed by Aristotle were three. The first and normal figure is when the middle term is the subject of the major and predicate of the minor. In the second, the middle term is the predicate of both, and in the third the subject of both. The fourth, which is reputed to have been introduced by Galen, and is largely dropped by logicians as an awkward and useless inversion of the first, occurs when the middle term is made the predicate of the major, and subject of the minor premise. Taking S, M, and P, respectively, for minor, middle, and major terms, the figures would be represented thus: Figures of Aristotle.

1st Fig.	2d.	3d.	4th
M P.	P M.	M P.	P M.
S M.	S M.	M S.	M S.
S P.	S P.	S P.	S P.
Sub. Præ.;	Tum Præ. Præ.;	Tum Sub. Sub.;	Tum Præ. Sub.

8. Of the eleven valid Moods, some are Invalid in one figure which are valid in another. Thus A E E would be valid in the second figure, as, Valid and Invalid Moods.

"All men are mortal,
No angels are mortal,
∴ No angels are men."

But in the first figure, it would involve illicit process of the major term. Thus:

"All birds are animals,
No reptiles are birds,
∴ No reptiles are animals."

The only valid moods in the first figure are A A A, E A E, A I I, E I O. As this is the figure into which the normal syllogism falls, logicians have usually unfolded the principles which govern the syllogism primarily with reference to that, and have devised ways of converting syllogisms in the other figures into it, and subjecting them to its tests. The canons which have been presented, however, apply immediately to the syllogisms in all the figures.

9. As is their wont, logicians have wrought out mnemonic lines in Latin to designate the valid moods and syllogisms in the several figures, with the modes of reducing the subordinate figures to the first.

Figure 1. { bArbArA, cElArEnt, dArII, fErIOque prioris.

Figure 2. { cEsArE, cAmEstrEs, fEstInO, bArOkO (or fAkOrO), secundæ.

Figure 3. { tertia, dArAptI, dIsAmIs, dAtIsI, fElAptOn, bOkArdO (or, dOkAmO), fErIsO, habet, quarta, insuper addit,

Figure 4. { brAmAntip, cAmEnEs, dImAarIs, fEsApo frEsIsOn.

In the foregoing lines the vowels signify the moods of the syllogisms respectively allowable in each figure. The initial letters b, c, d, f, denote that the syllogisms having them in the lower figures are to be reduced to the corresponding ones in the first. *m* indicates that in doing this, the premises are to be transposed, *s* and *p* that the proposition denoted by the vowel immediately preceding, is to be converted, *s*, simply *p*, *per accidens*, *i. e.* by limitation of quantity from universal to particular.

Explanation of Mnemonic Lines.

10. A slight examination of the three first figures —and for practical purposes the fourth may at present be passed by—will show that, in the First Figure, the minor premise must be affirmative in order to escape illicit process of the major term or negative premises, and that consequently the major premise must be universal in order to distribute the middle term. The Second

Limitations upon the several figures.

Upon the 1st.

Upon the 2d.

Figure can prove only negatives, because the middle term, being a predicate in both premises, requires at least one negative premise to distribute it. The Third Figure yields only particulars, because the major and minor terms, being both predicates, can only be distributed by having their respective premises negative. But only one of these can be negative, and if either be so it must be the major, for if it be the minor, it will make the conclusion negative, and thus distribute the major term, which, in this case, would be undistributed in the premises—thus bringing in illicit process of the major.

Upon the 3d.

11. It must, however, be remarked, that these properties of the several figures will be greatly modified in the case of the judgments in U and Y, which afford distributed affirmative predicates, and therefore cure all faults of the syllogism arising from the non-distribution of affirmative predicates. Inasmuch as it does not appear from the mere form of expression that any affirmatives distribute their predicates, it is always presumed that they do not, unless proved by other evidence. The analysis of the normal syllogism and its properties is therefore

Exceptions to the foregoing in the case of premises U and Y.

conducted on this presumption. But if the judgments usually classed as A and I, can in any case be shown to be U and Y in the syllogism, then neither of the foregoing limitations in respect to the several figures will hold. Thus, with these substitutive judgments, as premises, the first figure may have a negative minor without either illicit process or negative premises. Take the example,

U. "All men are (all) rational animals,
(Negative Minor.) E. No angels are men,
∴ No angels are rational animals."

Again,

(Particular Major.) "Some poets have genius,
Y. Some men are (all the) poets,
∴ Some men have genius."

Again in the second figure,

U. "Rational animals are men,
A. Poets are men,
(Affir. Conclusion.) ∴ Poets are rational animals."

Also in the third figure,

"All men are mortal,
U. All men are (all) rational animals,
(Universal Con.) A. ∴ All rational animals are mortal."

This is the proper formula of the Inductive Syllogism, which naturally falls into the third figure, and could not, aside from a

Formula of Inductive Syllogism.

substitutive judgment, yield a universal conclusion. Thus:

"X Y Z, are ruminant,
X Y Z, are (as good as) all horned animals,
∴ All horned animals are ruminant."

Sect. IV.—Maxims by which different logicians have applied the principles of Identity and Contradiction to the Syllogism.

Genus Predicated of Species.

12. Most of these are founded on the principle that, in a normal judgment, the Genus is predicated of the Species, and therefore that the extension of the subject is included in that of the predicate.

Aristotle's Dictum.

A. First among these maxims is the celebrated *Dictum* of Aristotle, that whatever can be predicated affirmatively or negatively of any class or term distributed, can be predicated in like manner of all and singular the classes or individuals contained under it. This is self-evident. Whatever can be affirmed or denied of all men, can be affirmed or denied of whatever is contained under the class man. This maxim is directly applicable to, and illustrated by the First Figure. Thus:

Directly applicable to First Figure.

"All men are mortal,
Poets are men,
∴ Poets are mortal."

Here mortal, being affirmed of the genus man, is also affirmed of the species poets included under it,

"No men are brutes,
Poets are men,
∴ They are not brutes."

Here, what is denied of the higher class, is also denied of the lower class or species included in it.

B. An equivalent maxim is that founded on the relation of Whole and Parts, that what may be affirmed or denied of a whole (in extension), may be likewise of its parts, *i. e.* what is predicated of a genus may be predicated of the species and individuals, or the parts composing it. *Pars partis est pars totius.*

Whole and Parts.

C. To the same effect is the maxim *contentum contenti est contentum continentis.* Men, the content of biped, is also the content of animal, which contains biped.

D. Kant's formula is, *nota notæ est nota rei ipsius.* This probably has reference to construing and testing Syllogisms according to the Intension of the terms. To this some of the fore-

Intensive Syllogisms.

going maxims apply, but in a reverse order, since the whole of extension increases as the whole of intension decreases. Therefore, in the Intensive Syllogism, the term of least extension, *i. e.* the minor, becomes the greater whole, and so in effect the major. Thus the Syllogism according to extension,

"All conquerors are brave,
Cæsar was a conqueror,
∴ He was brave,"

according to intension would be construed thus:

"Cæsar was a conqueror, *i. e.* had the mark or attribute of one,
Conquerors are brave, *i. e.* have the mark of bravery,
. He had the mark of bravery (was brave)."

Conclusion the same construed by Extension and Intension.

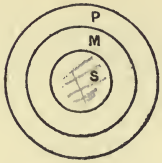
Construed either way, the connection of the same conclusion with the same premises, is equally certain and necessary. Sometimes the Extension, sometimes the Intension, is more prominent in the mind of the thinker.

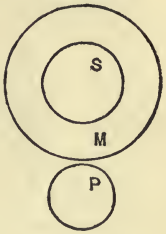
Illustration by Diagrams.

13. The relation of the several terms of the Syllogism to each other has often been exhibited to the eye by Circular Diagrams. Thus the Syllogisms of the several figures may be exhibited.

Barbara. **Celarent.**

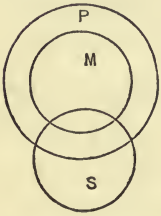
1st Figure.

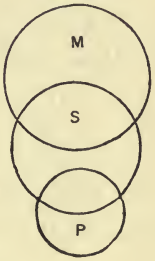




Darii. **Ferio.**

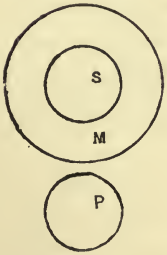
1st Figure.

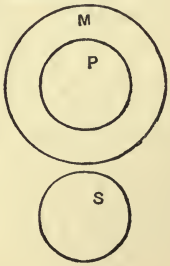




Cesare. Camestres etc., etc.

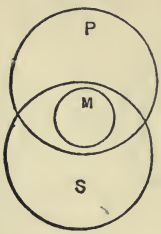
2d Figure.



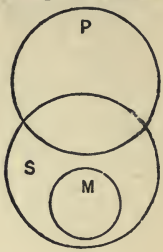


3d Figure.

Darapti.



Felapton, etc., etc.



For a fuller view of the different schemes of Syllogistic Notation, see Appendix B.

Sect. V.—Unfigured Syllogism.

How Figure disappears.

14. Before leaving this subject, it is proper to call attention briefly to a mode of analyzing the Syllogism introduced by Hamilton, which dispenses with Figure altogether. After the explicit quantification of both terms of a judgment, the relation between them may be expressed by the sign of equality, and either of them may become indifferently subject or predicate. In this way Figure disappears. If we say

"Men are rational,
Negroes are men,
∴ Negroes are rational."

we may more explicitly, though awkwardly, state our meaning thus;

"All men are = some rational.
All negroes are = some men.
∴ All negroes = some rational."

And it is obvious that the terms of either or all of these judgments may be transposed, without impairing the sense or reasoning. Thus:

"Some rational = all men.
Some men = all negroes.
∴ Some rational = all negroes."

All other figures may be similarly reduced. It is thus apparent that the Unfigured Syllogism expresses nakedly the essential principle which underlies reasoning in all the Figures.

Sect. VI.—Hypothetical Syllogisms.*

15. These are syllogisms in which the reasoning

* The use of the terms "hypothetical" and "conditional," as applied to judgments and syllogisms, varies with different logicians. Some use the word hypothetical to denote the genus, of which they make conditional and disjunctive the species. Others make conditional the genus, which includes hypothetical and disjunctive as species. That is, different writers make the words hypothetical and conditional change places.

turns upon the Hypothesis in a hypothetical judgment. A syllogism may contain hypothetical judgments in which the reasoning does not turn upon the hypothesis, but simply retains it as one of the terms of the conclusion. Thus:

Reasoning Turns on the Hypothesis.

Otherwise the Syllogism is Categorical.

"Every man is either a hero or a coward,
A. B. is a man,
∴ A. B. is either a hero or a coward."

'The books of Scripture are entitled to reverence, if its authors are not impostors,
The prophecies are books of Scripture,
Therefore the prophecies are entitled to reverence, if their authors are not impostors."

Such syllogisms are categorical.

16. But when the Reasoning turns on the Hypothesis, the Syllogism is Hypothetical, and becomes either Conditional, Disjunctive, or Dilemmatic, according as the Hypothetical Judgment on which it is founded, falls into one or the other of these classes. In these syllogisms the hypothetical judgment forms the major premise: one of its members affirmed or denied the minor—and the consequent affirmation or denial of some other member forms the conclusion. Thus:

Hypothetical Syllogism Described.

"Major. If rains are plenty, the crops are plenty,
Minor. The rains are plenty,
∴ The crops are plenty."

Sect. VII.—Conditional Syllogisms.

Conditional Judgments are founded on the principle of sufficient reason, otherwise called Reason and Consequent.

Grounded in Reason and Consequent.

17. The nature of the Conditional Judgment thus being, that on the ground of Reason and Consequent, if the antecedent is true the consequent is true, it follows;

A. That, if the Antecedent be affirmed in the minor premise, the Consequent must be affirmed in the conclusion.

Laws of Conditional Syllogism.

B. If the Consequent be denied, the Antecedent must be denied, since, if the latter were true, the former would be so likewise.

C. If the Antecedent be denied or the Consequent affirmed, no conclusion follows, for the latter may be true or the former false on other grounds. Of these the following are examples:

Examples.

A. If A is B C is D, A is B, ∴ C. is D.
B. If A is B C is D, C is not D, ∴ A is not B.
C. { If A is B C is D, A is not B, ∴ no conclusion.
  { If A is B C is D, C is D, ∴ no conclusion.

The fallacy of any inference in the cases under C, will appear more plainly from concrete examples. Thus, if we deny the antecedent, the following example will show that nothing follows.

Fallacies illustrated.

"If James is a drunkard he is unfit for office,
He is not a drunkard,"
∴ Nothing can be inferred.

So likewise from affirming the consequent nothing follows. Thus:

"If the people are virtuous they will establish schools,
They will establish schools,"
∴ No inference is warranted.

No fallacy is more common than that of drawing inferences in such cases.

Sect. VIII.—Disjunctive Syllogisms.

18. These are founded on the principle of Excluded Middle. Of two Contradictories, one must be true and the other false. There is no other alternative, no middle ground. Genuine disjunctives are mutually exclusive. That is, each member excludes the others. Whichever is true, the others are false. If either be false, some one of the others is true. Thus, "it is either Spring, Summer, Autumn, or Winter."

Rest on law of Excluded Middle.

Either of these excludes the others. Whichever is true, the others are false. Whichever is false, some one of the others is true. Hence with a disjunctive major;

Laws of the Disjunctive Syllogism.

First. If either member of it be affirmed in the minor, the other members are false. Thus:

"Men are either angels, brutes, or rational animals.
They are rational animals,
∴ They are neither angels nor brutes."

This is what the logicians call *modus ponendo tollens.*

Second. If, in the minor, either member of the major be denied, then some one of the other members is true. Thus, in the preceding example, if in the minor we say, "Men are not angels," it follows that they are either brutes or rational animals. This is *modus tollendo ponens.*

Disjunctives turned into Conditionals.

19. It is proper to repeat that a Disjunctive may be turned into a Conditional by taking the contradictory of one of its members for the antecedent. "It is either Spring or Summer," is the same as "if it is not Spring it is Summer." Increasing the members thus: "It is either Spring, Summer, Autumn or

Winter"—we get by conversion, "if it is not Spring, it is either Summer, Autumn, or Winter."

Sect. IX.—The Dilemma.

20. The Dilemma is a syllogism having a Dilemmatic Judgment for its Major Premise, with a Minor so affirming or denying some member or members of the major, as to lay the foundation for an inference. As this judgment is a combination of the conditional and disjunctive, so the Dilemma partakes of the characters of the conditional and disjunctive syllogism. The major premise of the dilemma may be of various forms, each capable of different minor premises, and so furnishing a ground for different conclusions.

Dilemma Defined.

A. The Major Premise may consist of one Antecedent with a Disjunctive Consequent. If A is B, either C is D or E is F. Affirm the Antecedent, A is B, and the Disjunctive Consequent, either C is D or E is F, follows. Deny the Consequent wholly, and the Antecedent must be denied. If neither C is D nor E is F, then A is not B. If, however, the Consequent be denied only disjunctively nothing can be inferred, for if either member of the Consequent be

Different forms of the Dilemma. One Antecedent and a Disjunctive Consequent.

true, the Antecedent may or may not be so. As in pure conditionals, from the mere denial of the Antecedent or affirmation of the Consequent, nothing can be inferred.

B. There may be a Plurality of Antecedents in the major, all having one Common Consequent. If A is B, X is Y, and if C is D, X is Y.

Plurality of Antecedents and a Common Consequent.

In this case, if the Antecedents be wholly or disjunctively granted, the one Common Consequent must follow. For if either of the Antecedents be true, the Consequent is true. If the Consequent be denied, all the Antecedents must be denied. But from affirming the Consequent or denying either or all the Antecedents, nothing can be inferred.

C. There may be a Plurality of Antecedents in the Major, each with its own Consequent. In this case, if the Antecedents be affirmed wholly, the Consequents may be affirmed wholly. If the Antecedents be affirmed disjunctively, the Consequents may be affirmed disjunctively. From the denial of Consequents wholly or disjunctively, the Antecedents may, in like manner, be denied wholly or disjunctively. But from any denial of the Antecedents or

Plurality of Antecedents, each with its own Consequent.

affirmation of the Consequents, nothing can be inferred.

"If men are virtuous they are wise,
And if they are vicious they are unwise;
But they are either virtuous or vicious,
∴ They are either wise or unwise."

Or denying the Consequent disjunctively,

"But either they are not wise or they are not unwise,
∴ Either they are not virtuous or not vicious."

That affirming the Antecedents or denying the Consequents wholly, would lead to a corresponding affirmation of Consequents or denial of Antecedents respectively, appears in the following example:

"If A. B. is diligent he will prosper,
And if C. D. is wise he will be diligent,
But A. B. is diligent and C. D. is wise,
∴ A. B. will prosper and C. D. will be diligent."

In like manner the denial of both Consequents involves the denial of both Antecedents.

Some Logicians, as Whateley, exhibit that alone as the only true Dilemma which has a plurality of Antecedents in the Major, and a disjunctive Minor.

Restriction of the Dilemma by some Logicians.

21. The Dilemma has been named the *Syllogis-*

mus Cornutus, or Horned Syllogism, because it confronts an adversary with two assumptions or arguments, on which it tosses him as on horns from one to the other, each being equally fatal to him. Hence the common phrase, "Take which horn of the Dilemma you will, it is equally fatal to you." Thus: Horns of the Dilemma.

"If things are what we can help, we ought not to fret about them, and if they are what we cannot help, we ought not to fret about them. But all things are either what we can or cannot help. ∴ They are what we ought not to fret about."

22. The names Trilemma, Tetralemma, Polylemma have been sometimes given to this sort of Syllogism according to the number of members or horns, if they exceed two. Thus: Trilemma, Tetralemma, etc.

"If A is B, X is Y, and if C is D, X is Y, and if E is F, X is Y. But either A is B or C is D or E is F, ∴ X is Y," is a Trilemma.

23. The ultimate principles which determine the resolution of the Dilemma are those which determine the conditionals and disjunctives out of which it is formed. Ultimate principles.

Sect. X.—Incomplete Syllogisms.

24. In ordinary reasoning, it is seldom that the process is fully expressed in a completed Syllogism. One of the premises is often wholly, and the other partially unexpressed. A syllogism with one premise unexpressed is an Enthymeme. Thus:

Enthymeme.

"The Americans are a free people,
∴ They are happy."

Here the unexpressed Major premise,

"All free peoples are happy,"

is obvious. In this:

"Bankers are wealthy,
∴ A. B. is wealthy,"

The Minor premise,

"A. B. is a banker,"

is unexpressed.

25. Enthymemes, like Complete Syllogisms, often express the conclusion with "because," or other equivalent particles, between it and the premise. Thus:

In varied forms.

"A. B. and C. are unfit to vote because they cannot read."

The learner will readily complete such a Syllogism in regular form. Indeed the forms of En-

thymemes, occurring in ordinary speech, are innumerable. Thus:

"These men are good and therefore brave," etc., etc.

Sect. XI.—Complex Syllogisms.

26. Several Syllogisms may be combined and abridged, so that the conclusiveness of the reasoning shall be just as evident as if they were all fully expressed. Chief of this kind is the

SORITES,

Or chain-syllogism, in which a number of syllogisms in the First Figure are so combined, that the predicate of the first premise becomes the subject of the next, and so on, until, in the conclusion, the predicate of the last premise is predicated of the subject of the first. Thus:

Sorites defined.

"The Hindoos are Asiatics,
The Asiatics are men,
Men are rational animals,
Rational animals have body and spirit,
.. The Hindoos have body and spirit."

The conclusiveness of this may be represented thus:

27. The following principles control the Sorites.

Principles and laws of Sorites.

A. The several unexpressed propositions are respectively conclusions of each next preceding syllogism. Each of them becomes in turn the minor premise of the next following, as will easily appear by completing the several syllogisms.

B. All the intermediate expressed premises, therefore, between the first and the conclusion, are major. The first alone is minor.

C. Hence no premise except the first can be particular, for the first figure must always have a universal major in order to distribute the middle term.

D. Hence, again, no premise can be negative except the last; for a negative premise would make the conclusion negative, which in turn would become the negative minor premise of the next syllogism. This

has been shown, in the first figure, to beget illicit process of the major, and is not allowable.*

GOCLENIAN SORITES.

28. This is a form of the Sorites, so named because it was first invented or brought to view by Goclenius. It simply inverts the order of the premises as found in the common Sorites. Thus, if we take the example before given, it can be stated as follows: Inverted Sorites.

"Rational animals are composed of body and spirit,
Men are rational animals,
Asiatics are men,
The Hindoos are Asiatics,
∴ The Hindoos are composed of body and spirit."

In this form of Sorites, each preceding subject becomes the predicate of the next, until, in the conclusion, the predicate of the first premise is predicated of the subject of the last. The last premise alone may be particular, and none but the first can be negative.

* These conditions, however, are subject to any exceptions which might arise from substitutive judgments in any of the premises. So also of the Sorites in every form.

HYPOTHETICAL SORITES.

Hypothetical Sorites.

29. It is plain that a Sorites may be conditional as well as categorical. Thus·

If A is B, C is D,
If C is D, E is F,
If E is F, X is Y, but A is B,
∴ X is Y. (Modus ponens), or X is not Y.
∴ A is not B. (Modus tollens).

In regressive form thus:

If E is F, X is Y,
If C is D, E is F,
If A is B, C is D. But A is B, ∴ X is Y.
Or X is not Y. ∴ A is not B.

Direct Form.

If A B is virtuous, he is brave,
If brave, he is magnanimous,
If magnanimous, he will do noble deeds,
But he is virtuous, ∴ he will do noble deeds.

PROSYLLOGISM, EPISYLLOGISM AND EPICHEIREMA.

30. The different forms of complex Syllogisms comprise the modes in which separate syllogisms are combined into wholes of connected reasoning. In these the Sorites is rare. The Prosyllogism and Episyllogism are of constant occurrence.

The Prosyllogism is one whose conclusion furnishes a premise for the principal argument. The Episyllogism makes the

Prosyllogism.
Episyllogism.

conclusion of the main argument one of its premises.

"Useful studies ought to be pursued:

Prosyllogism.

Logic is a useful study (since it helps to think well),

Episyllogism.

∴ It ought to be studied, and (hence an educational course which omits Logic is deficient)."

31. Epicheirema denotes a Syllogism which has a Prosyllogism to establish each of its premises. Thus: Epicheirema.

"Man has a spirit, for he is rational,
And he has a body, for he fills space,
∴ Some thing that has a spirit has body."

This name is also applied sometimes in cases where there is a single Prosyllogism.

Polysyllogism is a combination of several syllogisms in one argument. The Sorites is one species of it. Polysyllogism.

CHAPTER VI.

APPLIED LOGIC—FALLACIES.

1. HAVING brought to view the fundamental laws of pure thinking, or principles of Formal Logic, as related to Conceptions, Judgments, and Reasonings, it remains that we now treat, as briefly as possible, of the application of these principles, first to the detection and avoidance of errors in thinking; and next, to the right conduct of the thinking process, when employed in the discovery of truth as pertaining to actual being. The former brings us to the doctrine of Fallacies, the latter of Method.* And first,

Transition to Applied Logic.

Fallacies.

Method.

SECTION I.—FALLACIES.

2. A Fallacy is any unsound or delusive mode of reasoning, which wears a specious appearance of being genuine, and thus often has power to impose upon men.

Fallacies defined.

* For a fuller exhibition of the difference between Formal and Applied Logic, the student is referred to the observations on this subject in Chap. I., Sect. IV.

3. Fallacies are divisible into Paralogisms and Sophisms. A Paralogism is a fault in reasoning unknown to him who employs it. A Sophism, or sophistical reasoning, is a faulty argument understood by him who employs it, and used for the very purpose of deceiving. It is proper to add, however, that these distinctions have no logical, whatever may be their moral significance, and that they are often overlooked by good writers who use the terms Fallacy, Paralogism, and Sophism interchangeably and indiscriminately.

Divided into Paralogisms and Sophisms. Definition of each.

Both have the same Logical force.

4. Fallacies are further divisible into Formal and Material. The former are those in which no conclusion follows from the premises, however there may be an appearance of it. These are all cases of more than three terms, Undistributed Middle, Illicit Process, Negative Premises, affirmative conclusion with either premise negative,*

Formal and Material Fallacies distinguished.

Instances of Formal Fallacies.

* It is important, however, to remember that many propositions, in form negative, are not so in the fact, because the force of the negative particle falls on the subject or predicate instead of the copula. Propositions are in reality negative only when

making any conclusion from particular premises, or a universal conclusion when either premise is particular, except when Substitutive Judgments furnish the necessary distribution of terms,* from denying

the real import of the copula is negative, so dividing the two terms from each other. Thus:

"He who has not enough is not really rich,
No miser has enough,
∴ No miser is really rich."

The minor premise is really equivalent to

"All misers are persons who have not enough,
∴ All misers are persons not really rich."

"No person who is not secure is happy,
No tyrant is secure = All tyrants are persons not secure,
∴ No tyrant is happy."

Where both premises are really negative such an experiment will not succeed.

"Vicious persons are not happy,
A and B are not vicious,
∴ No conclusion."

All attempts to transfer the negative particle to one of the terms here, will result in Four Terms, or Undistributed Middle, or in altering the meaning of one premise.

* Such an exception is the following:

"Some mortals are (all) men,
Some men are (all the) poets,
∴ All the poets are mortal."

the Antecedent or affirming the Consequent of a conditional; and from violating any of the canons of inference in Disjunctives and Dilemmas: inferring A from A, or O from O, by conversion, etc., etc. These have been developed already under Formal Logic, and belong properly to it. They are vices in the very form of thinking, whatever be the premises or conclusion. They do not, indeed, belong to real thought, but only to the counterfeits which simulate it. They enter into Applied Logic only as principles of Formal Logic which are applied to detect vices in reasoning about matters of actual being. Indeed, they would hardly need to be introduced here at all, were they always put in such phrase as to be palpable. If apparent, the invalidity of the argument in which they occur is self-evident. They are, however, very apt to be disguised under equivocal or vague expressions; or, for other reasons, to elude the notice of those concerned. On this account they require to be noticed in Applied as well as in Formal Logic.

Why introduced in Applied Logic.

Often disguised.

5. Material Fallacies are such as occur when there is no fault in the reasoning process, and the conclusion does follow from the pre-

Material Fallacies Defined.

mises. Hence called Material, because they lie not in the form, but the matter of the Syllogism. Is it asked, how is a fallacy possible here? The answer is, 1st, that a premise may be unwarrantably assumed, or 2d, the conclusion may be irrelevant. It may fall short of what the reasoner intends or professes to prove. The technical name of this latter is *Ignoratio Elenchi*—ignorance of the proof of the real issue, the contradictory of your adversary's proposition which you undertake or assume to demolish. This is a fallacy of very frequent occurrence. It is a common defense of criminals to allege that they were insane; and to attempt to prove this by showing that they acted very unreasonably! But this is not to the purpose, for if it were, all criminals would be maniacs, and guilt would be impossible. So it is a frequent and wicked practice of this fallacy or sophism, to arouse the passions of the tribunal appealed to in regard to the atrocity of an imputed offense, instead of proving it to have been committed by the accused.

Groundless premise or irrelevant conclusion.

Ignoratio Elenchi.

Examples.

6. To this head may be referred various arguments which logicians have been accustomed to con-

trast with *argumentum ad rem, i. e.* to the point. Such is *argumentum ad verecundiam*, or appealing to the feelings of reverence for certain persons or objects, instead of proving the point in hand: *argumentum ad ignorantiam*, assuming that your position is correct unless your adversary can evince the contrary: or it is sometimes used to denote any sort of sophism which imposes on men's ignorance: *argumentum ad populum*, which is very much akin, being addressed to the passions and prejudices rather than the intelligence of the people; and finally *argumentum ad hominem*, an appeal to the practice, principles, or professions of an adversary, as confirmatory of our own position or fatal to his.

Argumentum ad rem. Ad verecundiam. Ad ignorantiam Ad populum. Ad hominem.

This argument is legitimate so far as concerns an adversary, and for the purpose of silencing him. If understood to be limited to this, it is not objectionable. So our Saviour often employed it to silence the cavils of the Pharisees and other adversaries. It is illegitimate when employed as if it established any proposition absolutely, or were binding upon any besides those whose personal opinions and conduct thus

How far valid.

make against their positions; or even upon them, after they renounce such opinions and conduct.

7. The other sort of material fallacy by the unwarrantable assumption of a premise, has some forms that have been signalized by corresponding names. Chief among these is,

Petitio Principii or begging the question, which is the unwarrantable virtual assumption of the thing to be proved, or of that by which it is to be proved, without proving it, in the course of the argument. Thus, if one undertake to show that a given tariff will be beneficial because it will promote the public wealth, without proving this latter, he perpetrates a *petitio principii*. The most deceptive form of this fallacy is,

Petitio Principii.

Arguing in a circle—*argumentum in circulo*—in which the conclusion is virtually used to prove the premise, thus going in a circle which returns upon itself, from premise to conclusion and from conclusion to premise. To argue that certain men are good because they belong to an excellent party, and that this party is excellent because it includes such worthy members, is to argue in a circle. Some demonstrate

Arguing in a Circle.

the immortality of the soul from its simplicity, and then its simplicity from its immortality.

8. *Non causa pro causa* assumes that to be a cause which is not a cause. Foremost among these is the fallacy of *post hoc ergo propter hoc*, taking a mere antecedent of an event to be, as a matter of course, its cause. As if, because night precedes day, it were therefore the cause of day, or because civil war in the United States preceded the continental war between Austria, Prussia, and Italy, it were therefore the cause of that war.* Non causa pro causa.

9. An assumption analogous to this is the taking of *non tale pro tali*, assuming a resemblance without proving it. Thus, "the season is favorable to apples because peaches are abundant," implying such a resemblance between these two kinds of fruit, and the requisites to their growth, as warrants such an inference. "All other Non tale pro tali.

* Notwithstanding the elaborate efforts of Mill, Brown, and others to prove that cause is only antecedent or invariable antecedent, the intuitive judgment of the human race is well voiced in the following words of Cicero.

"Causa est ea quid efficit id cujus est causa. Non sic causa intelligi debet, ut, quod cuique antecedat, id ei causa sit, sed quod cuique efficienter antecedat."—Quoted in Bowen's Logic, p. 306.

religions are delusions. Therefore Christianity is a delusion."

Sect. II.—Fallacies partly formal and partly material.

Semi-Logical Fallacies.

10. By far the most numerous and misleading class of Fallacies, are those styled by Whateley "semi-logical." This term has been criticized as absurd, as if there were no conceivable medium between a Fallacy purely logical, or non-logical. But whatever may be said of the term, he employs it to denote a reality which no other term adequately denotes. It denotes the class of Fallacies arising from the ambiguous use of terms in reasoning, or in the syllogism.

The term Explained.

An Ambiguous Term = Two Terms.

11. An Ambiguous Term is equivalent to Two Terms; consequently, if either of the three terms of a syllogism be ambiguous, it amounts to bringing a fourth term into it. But when there are four terms there can be no conclusion. We see then how this Fallacy of Ambiguous Terms is partly material and partly formal. In order to detect the ambiguity, we have to look at the

How Semi-Logical.

matter of the syllogism as contained in the meaning of its terms. So far it is material. When the ambiguity is detected, the fault which gives rise to the fallacy, is shown at once to be formal, because the syllogism is loaded with four terms which are incompatible with any conclusion. It is true that, at bottom and in essence, this fallacy is formal. But the discovery of it requires examination of the matter embraced in the syllogism. Thus:

"Feathers are light,
Light is contrary to darkness,
∴ Feathers are contrary to darkness,"

is a syllogism in reality with four terms, two of which are words spelt with the same letters, but of different meanings. This difference of meaning must be ascertained in order to expose the fallacy.

12. Fallacies of this description are far the most specious and numerous of all, and are as various as the various causes or kinds of ambiguity in language. We will call attention to a few of the more prominent that logicians have been accustomed specially to designate.

Such Fallacies Specious.

13. The fallacy of Division and Composition. In this the middle term is taken dividedly or distributively in one premise, and collectively in the other. Thus:

Division and Composition.

"All these persons are a crowd,
A. and B. are some of these persons,
∴ They are a crowd."

Here these persons are taken collectively in the major, and otherwise in the minor.

"Five is one number,
Three and two are five.
∴ They are one number."

This is composition in the major and division in the minor.

14. This fallacy is of constant occurrence in connection with the word "all," which, in the peculiar idiom of our language, affords great facilities for it. First, as in the examples given above;

Fallacy of the word "all."

"All these soldiers are an army,
All these soldiers are individual persons,
∴ Individual persons are an army."

Here in the major "all" is taken collectively, in the minor distributively.

But the greatest liability to an ambiguous or non-natural sense of the word "all," is where it is the subject of a negative judgment, in which case it is nevertheless impossible to deny the predicate of "all" the subject. Thus:

Of "not all."

"Not all men are poets, or
All men are not poets,"

is equivalent to

"Not every man is a poet, or
Some men are not poets."

Sometimes there is danger of construing "not all" as equivalent to none, whereas it only amounts to "not some." This is well illustrated by Whateley in the following example:

"If all testimony to miracles is to be admitted, the Popish legends are to be believed; but the Popish legends are not to be believed; therefore *no* (for "not all") testimony to miracles is to be admitted."

It is important to be on our guard against fallacies arising from ambiguities in this pregnant monosyllable.

15. A very ensnaring form of ambiguous middle is known as *Fallacia Accidentis*, or *a dicto secundum quid ad dictum sim-*

Fallacia Accidentis.

pliciter, and vice versa, *i. e.* of using the middle term considered with reference to some of its accidents in one premise, and with reference to its mere essence in the other.

"The covering of sheep is what we wear,
Undressed wool is the covering of sheep,
Undressed wool is what we wear."

Again:

"Government is a blessing,
The most cruel despotism is a government,
∴ Therefore it is a blessing."

Fallacy of Etymology.

16. A very common form of ambiguous middle is that founded on Etymology, or the assumption that derivative, paronymous, or conjugate words have the signification of their roots, and compounds of their originals. It is true indeed, that the meaning of words sometimes remains unchanged through all these variations. Sometimes the changes of meaning are slight, but, for that very reason, all the more liable to be overlooked and to gender fallacies. Thus:

"Projectors ought not to be trusted,
This man has formed a project,
.. He ought not to be trusted'

"Artful persons should be shunned,
A. B. is a great artist,
∴ He ought to be shunned."

"Truth is derived from to trow, *i. e.* believe,
But belief is variable,
∴ Truth is variable, *i. e.* not immutable."

17. Analogous to this is the Fallacy of Interrogations, sometimes called *Fallacia Plurium Interrogationum*. This is practiced when, under one question in form, by ambiguity of meaning, more than one question in reality is put, so that the person questioned is entrapped, whatever answer he may give. This is a trick frequently practiced by examiners of witnesses. Lawyers are peculiarly prone to it. They put ambiguous and embarrassing questions, and then with great show of sincerity and fairness, insist on a categorical *yes* or *no* for answer, as if to refuse such an answer would imply a lack of truthfulness, when in fact, such a categorical answer must be false or inadequate, owing to the ambiguous implications of the interrogation. Fallacy of Interrogations.

So the attempt is often made to ensnare or deceive, by a false assertion or implication, in a question so put as to imply that it is False Implications.

beyond dispute. No better instance of this can be found than the celebrated question of Charles II. to the Royal Society, "Why a dead fish does not, though a live fish does, add to the weight of a vessel of water in which it is placed?" This was put with such apparent assurance that some of the philosophers were, for the time, deceived, and busied themselves in seeking an explanation of the fact, while they omitted to inquire if it was a fact. So, many an innocent person has been entangled and led to criminate himself, being for the moment unmanned and thrown off his guard, by the very audacity with which such questions were put to him as these: "How long since you left off drinking, swearing, back-biting," etc.? No duty is more incumbent on courts than that of protecting witnesses and parties against such injustice.

Example.

18. Quite similar to this is the demand often made upon witnesses by examiners, not only for a Clear, but for a Distinct and even Adequate Cognition (see chap. II., Sects. 9, 30) implying that their testimony is to be suspected, unless, besides certainty as to the object testified about, they can also give its marks. Thus, if a witness testifies that

Demand for Distinct and Adequate Cognition.

Example.

a certain signature or manuscript is in a given man's hand-writing, it is quite common to insist that he should give some of the marks or distinctive peculiarities by which he distinguishes the chirography in question. The same thing is often done in examinations for the purpose of identifying persons, places, and other objects. The fallacy of all this, so far as it implies distrust of the testimony of those who are unable to give the marks, is palpable. In general it is only the few experts, in each department, who, besides knowing objects with certainty, can give the distinguishing marks or definitions of them. There are few things that we know with more certainty than the different hand-writings with which we have been familiar. There are few matters in respect to which those who have not made it a subject of special study, will more certainly and egregiously blunder, than in attempting to give the marks which distinguish the chirography of different persons. So with other things. Nothing would sooner nonplus such questioners themselves than to exact of them a logical definition of words, or the marks of conceptions, with which they are perfectly familiar, and which they constantly use with substantial accuracy.

Fallacy Exposed.

19. Another fallacy is the Over-estimation of Probabilities, *i. e.* of the degree of belief which ought to be produced by evidence less than certain—especially of supposing that a plurality of probabilities necessarily strengthen each other. *A single probability of any uncertain event is ascertained by dividing the number of chances favorable to the event by the total number of chances.* Thus the probability that a person blindfolded will take a black ball out of an urn containing 10 white and 2 black balls is $\frac{2}{12}$ or $\frac{1}{6}$.

Over-estimation of Probabilities.

20. "To find the chance of the recurrence of an event already observed, *divide the number of times the event has been observed, increased by one, by the same number increased by two.* If an inlander coming to the sea, observed the phenomenon of the tide ten times in succession, the chance to him that at the next period the tide would again rise would be $\frac{10}{10}+\frac{1}{2}=\frac{11}{12}$; or 11 to 1. Every certainty is represented by a unit, as has been shown; and so many units are added to the possible cases (denominator of the fraction) as there have been events, and so many to the favorable cases (numerator) as there have been favorable events. 'Or, if we represent,' says M. Quetelet, 'the number of times that the

event has occurred by a similar number of white balls that we throw into an urn, adding also one other white ball and one black ball, the probability of the reproduction will be equal to that of drawing a white ball.'

"In order to calculate the probability that an event already observed will be repeated any given number of times, the rule is, to *divide the number of times the event has been observed, increased by one, by the same number increased by one and the number of times the event is to recur.* Thus, if the tide had been observed 9 times, the chance that it would recur ten times more would be $\frac{9}{9+1} \frac{1}{1} = (\frac{10}{20}) = \frac{1}{2}$. 'This is the same thing as if each reproduction of the observed event corresponded to putting a white ball in an urn where there were already, before commencing the trials, a white ball and as many black balls as it is supposed that the event observed should re-occur times.'"—*Thomson's Laws of Thought.*

21. If two or more probabilities are independent of each other, they do afford mutual support. But if otherwise, if they are probabilities of probabilities, they weaken each other. If the credibility of a witness be $\frac{3}{4}$ so far as his ability to observe aright

When they strengthen and when they weaken each other.

and know the facts is concerned, $\frac{3}{4}$ so far as his veracity is concerned, then the total probability of his telling the truth is $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$, unity being the representative of certainty.

22. If, however, the probabilities are mutually independent, they strengthen each other, and as they increase in number and force, they may come short of certainty by only an infinitesimal distance. Thus, if the probability that A. B. committed a given murder be strong, 1, from certain money belonging to the victim being found in his possession; 2, from his boots fitting tracks found near the place of murder; 3, from blood on his clothes; 4, from a piece of knife-blade found in the head of the murdered body fitting precisely the broken blade of a bloody knife found in the pocket of the suspected person; it is clear that all these separate probabilities confirm each other, and together fall only short of apodictic proof. In this case, the mode of computing the absolute probability, is to subtract each separate probability from unity, which gives the probability of the opposite event, or of failure arising from each several cause. But as these several probabilities of the opposite event weaken each other, or are probabilities of probabilities, the

entire probability of it is ascertained by multiplying the separate ones together. This product subtracted from unity will give the probability of the original event in question, of which this is the opposite.* Thus in the example just given; let the first probability be $\frac{1}{2}$, the second $\frac{1}{3}$, the third $\frac{1}{4}$, the fourth $\frac{3}{4}$. Subtracting each of these from unity, and multiplying them together, we have $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{1}{4}=\frac{6}{96}=\frac{1}{16}$, which, subtracted from 1, gives $\frac{15}{16}$, as the probability that the suspected person was the real murderer—a probability sufficient to neutralize all reasonable and practical doubt.

23. Strictly, however, this and all positive directions touching the calculation of probabilities, belong to the doctrine of Method. It comes in here very naturally, however, in connection with the correlate fallacy.

Strictly belongs to Logical Method.

* "As, in the case of two probable premises, the conclusion is not established except on the supposition of their *both* being true, so in the case of two (and the like holds good with any number) distinct and independent indications of the truth of some proposition, unless *both* of them *fail*, the proposition must be true; we therefore multiply together the fractions indicating the probability of *failure* of each,—the chances *against* it; and the result being the total chances against the establishment of the conclusion by these arguments, this fraction being deducted from unity, the remainder gives the probability *for* it."—*Whateley's Logic*, Book III., 15.

Ambiguity Fictæ Universalitatis.

24. A source of ambiguity, not only in the middle, but other terms, which ought not to be overlooked, although the means of guarding against it, will more fully appear under the head of Induction, has received the name *fictæ universalitatis* ∴ *i. e.* of a groundless inference from a few cases to all cases. This is among the most common forms of delusive and fallacious reasoning. Common Examples of this are, that Friday is an unlucky day, because some enterprises begun on that day have suffered disaster: that an epidemic is raging, when only the fewest cases of disease have appeared: that hemorrhage of the lungs is always fatal, because it is often so: that all men are knaves because so many are: that the whole community are of a given opinion, because A. B. and C. have expressed it. Out of such fictitious universals arise Syllogisms like the following:

Examples.

> "Men love to be humbugged,
> The President of the Bible Society is a man,
> ∴ He loves to be humbugged."

Sources of Ambiguous Middle.

25. The sources of ambiguous middle are as numerous and varied as the sources of ambiguity in language itself. Their de-

tection and correction belongs rather to rhetoric, grammar, or philology, than to logic. We have no room to pursue it further here. Those who desire to see it unfolded at greater length, may consult the chapter on Fallacies in *Whateley's Logic* with interest and profit.

26. It only remains that in concluding the subject of Fallacies we present some specimens of

Sect. III.—Logical Puzzles.

In inventing which the intellectual activity of past times exerted itself, for lack of worthier objects. These have been bequeathed to succeeding generations to task their subtlety, and at once amuse and perplex students in their leisure hours. This however has not been the worst of it. They have gone far to countenance the impression that Logic, instead of being a genuine or useful science, is little better than a kind of jugglery and legerdemain, for working up seeming demonstrations of manifest absurdity and falsehood.

Logical Puzzles more ingenious than useful.

27. The Dilemma is a favorite instrument for this sort of logical sleight of hand. A sly fault in some member of

Use of the Dilemma for this purpose.

its complex parts affords the facile opportunity for it, because it is so readily unobserved. The standard examples we are about to quote from the books, will illustrate this.

28. "In sifting a proposed Dilemma," says Krug, "we are to look closely to the three following particulars:—1. Whether, in the Sumption,* the Consequent is a legitimate inference from the Antecedent; 2. Whether the Disjunction in the Consequent is complete; 3. Whether, in the Subsumption,† the Disjunct Members are properly sublated. The following Dilemma is faulty in each of these respects.

Krug's rules for sifting Dilemmas.

"If Philosophy be of any value, it must procure for us power, riches, or honor.

"But it procures neither of them. Therefore," etc.

"Here, 1, the inference is wrong, as Philosophy may be worth something, though it does not secure any of these external advantages; 2, the Disjunction is incomplete, as there are other goods, besides the three here enumerated; 3, the Subsumption is false, as Philosophy has often been the means of procuring these very advantages."

Solution.

* Major premise. † Minor premise.

29. Analogous to this is the old quibble to disprove the possibility of motion, which also throws up the horns of a dilemma. Thus: Puzzle about Motion.

"If Motion is possible, a body must move either in the place where it is, or in a place where it is not.

"But a body cannot move in a place where it is; and of course it cannot move where it is not.

"Therefore, motion is impossible."

The Major Premise or Sumption is false and involves a Material Fallacy. The true statement is that, if motion is possible, a body must move *from* the place where it *is* to a place where it will be. This removes every appearance of a puzzle. The Major Premise is false except with regard to one indivisible moment. But that is irrelevant to motion, which in its nature requires time, while the cognition of it supposes memory. Solution.

30. To the same complexion comes the famous old Puzzle named *Ignava Ratio*, *i. e.* the argument for inaction, because events being predetermined or otherwise fixed, all effort to alter them, or to attain what is desirable and avert Ignava Ratio.

what is evil, is unavailing. Cicero thus states it as urged against calling in medical aid in sickness:

"If it is fated that you shall recover from the present disease, then you will recover whether you call in a physician or not. If it is fated that you shall not recover, then, with or without a physician, you will not recover.

"But either the one or the other of these is fated.

"Therefore, it will be of no use to call in a doctor."

The obvious fallacy here, to look no deeper, lies in the fact, that the calling in of the doctor and using his prescriptions, may be the very means by which it is ordered that recovery shall take place; hence the first member of the sumption or major premise is false. And so of all analogous cases.

Achilles and the tortoise.

31. The famous puzzle of Achilles and the tortoise, which so long baffled the logicians, aiming to prove, by logic, the logical absurdity, that the swiftest runner can never overtake the slowest, is put thus:

"The swiftest runner can never overtake the slowest, if the latter has ever so little a start. Suppose, for instance, that Achilles runs ten times as fast as a tortoise, and that the tortoise is one mile in advance at the outset. While Achilles is traversing this mile, the tortoise has advanced $\frac{1}{10}$th of a mile farther; before his pursuer has passed over this $\frac{1}{10}$th, the tortoise has advanced $\frac{1}{100}$th, and then, again, $\frac{1}{1000}$th, and so on

forever, always being some fraction, however small, of a mile in advance."

The sophism here is disguised under a false statement of the problem. The real question, when will Achilles overtake the tortoise? is kept out of sight, and another wholly different substituted in its place, viz., if the tortoise is at any given point ahead of Achilles, how far will it have gone when Achilles shall reach that point? This soon runs into infinitesmals which are practical zeros, and, even if theoretically infinite in number, really are all included in that finite length which Achilles will quickly get over, leaving the tortoise behind.

The Sophism exposed.

32. Other puzzles abound on which we have no room to dwell. It is the less necessary, as a careful application of the principles already laid down, will readily solve them. The propounding and solution of such quibbles may be a casual diversion, it cannot be a principal object of pursuit, in any science worth serious study.

Such puzzles have no chief place in Logic.

CHAPTER VII.

LOGICAL METHOD.

1. METHOD, *μεθόδος*, is the way by which we proceed to a given goal. Logical Method is the way of applying the principles of Logic to the discovery, confirmation, or elucidation of the truth.

Method Defined.

In order to this, it is necessary to determine the sphere and matter, the extension and intension, the objects and the qualities, with which we have to do. The former is accomplished by Logical Division, and the latter by Definition, which have been duly treated in their respective places, in the Chapter on Conceptions. To this we refer the student as sufficient for present purposes, while we pass to consider more especially the use of Reasoning in the search and proof of truth.

Proceeds by Division, Definition, and Reasoning.

2. It must not be forgotten that Logic does not give us the original facts, axioms, or first principles, which constitute the primary matter or groundwork of our knowledge. These are furnished by Intuition, either 1. Of the phenomena of consciousness, *i. e.* psychological facts: or 2. By sense-perceptions, *i. e.* of facts pertaining to the material and external world—or, 3. By supersensual intuitive truths, *i. e.* self-evident axioms: or finally, by testimony either spoken or recorded. Logic deals with the matter thus afforded in a two-fold way. 1. In the application of its principles to test and explicate what is contained implicitly in the matter so furnished by the intuitive faculties: 2. By guiding us in such use of our intuitive faculties, as shall be most effective for advancing our knowledge. According to the former, the laws of Conceptions, Judgments, and Reasonings show what is, and what is not, necessarily implied by the facts and truths given us from other sources. In the latter, it helps to guide our inquiries, observations, and experiments towards the search for and intuition of such facts as will tend to elucidate or decide questions in issue, thus

Logic not the Original source of Knowledge.

Its sources enumerated.

How Logic deals with the matter so furnished.

saving us the waste of our powers in irrelevant and fruitless investigations.

Section I.—Original and Derivative Sources of Knowledge.

3. Our Original Sources of Knowledge then are the Intuitive (including Self-Consciousness, Sense-Perception, Self-Evident, Supersensual truths), and Testimony. The Derivative are what we derive from them through the power of Discursive Thought, including Abstraction, Generalization, Conception, Judgment, Reasoning. Some add to these Memory, of whom some class it with the former faculties, some with the latter. It is unnecessary to discuss this question here. It is enough that Memory is not itself a direct source of knowledge intuitive or discursive. It simply keeps and reproduces what is known through the other faculties. Some questions too might arise, as to how far Testimony is an intuitive, or immediate source of knowledge. It is not our plan here to go far in the discussion of such extra-logical questions. They are to be relegated to psychology, except so far as may be essential to a due understanding of Logic or

Recapitulation of sources of Knowledge.

Memory.

its applications. It suffices for our present purpose that Memory, like the intuitive faculties, furnishes, inasmuch as it preserves, material for the discursive faculties, but is not itself discursive.

Memory involved in Testimony.

4. Memory is an essential element in nearly all Testimony. It is rare that any one bears witness simply to the cognitions of the present moment. Almost all testimony respects the past.

Importance of Testimony.

5. Testimony is a fundamental source of knowledge. All facts known to us beyond the narrow circle of our own experience, must be learned from Testimony. And our generalizations and reasonings would be extremely scanty for lack of material, without the results of the experience of other men, added to our own, and authentically reported to us.

Oral and Recorded.

Testimony may be either Oral, or Recorded in historical writings, monuments, mementoes, and tokens. The canons for distinguishing true testimony from false, and genuine from spurious, authentic from fictitious history, are manifold and easily accessible. To discuss them is aside of our present purpose and beyond our space.

Testimony of God in his Word infallible.

6. There is, however, one species of testimony that is wholly unique, and above the plane of all human witnessing. We refer to the testimony of God in his Word. This is absolutely sure and infallible, being the utterance of Him for whom it is impossible to err or to lie. It is the exclusive source and foundation of Christian Theology. It is absolutely true and authoritative. To unfold the rules for the correct interpretation of Scripture would be to trench on the sphere of exegetical theology.

Theology founded on the authority of God.

7. It is proper, however, to remark that the first principles of theology do not depend upon any process of reasoning, *a priori* or inductive, but upon the authority of God who declares them. In a qualified sense, the true process for ascertaining what the Scriptures teach may be viewed as inductive. In other words, it simply ascertains and compares the actual teachings of Scripture, instead of deciding *a priori* what they may and may not teach.

The laws of thought always applied.

8. The application of the laws of thought or principles of logic to the facts, that are always coming before us in an isolated and unorganized form, is constantly

made, consciously or unconsciously, by all men. The power to do it is one of man's chief prerogatives as compared with the brutes. To think at all is, either consciously or unawares, to conform to the laws of thought. All else called thinking only simulates and counterfeits it. But, in proportion as this application of the principles of logic becomes comprehensive and complete, in regard to any given department of facts or truths, it becomes a scientific view of them. Thus, a comprehension of the facts concerning life, in their mutual relations, their harmony and unity according to the necessary laws of thought, makes up the science of Physiology; of the phenomena of the soul, Psychology; of spatial quantity and relations, Geometry.

Unlogical is counterfeit thought.

9. Science then is not a mere knowledge of disjointed unreconciled facts or truths, but a knowledge of these facts as mutually related, harmonized, and unified, under all-inclusive principles and laws. But, in the sphere of actual being, of events or phenomena, to ascertain their laws and principles is commonly to ascertain their causes. Towards this state all knowledge tends in proportion as it tends

Science, what it is.

To find laws is to find causes.

to perfection. And this, not only in each particular department of inquiry considered by itself, but in the relation of them all to each other. They are more and more comprehended in their mutual relations and harmony, until they culminate in absolute unity in the Great First Cause, and the Infinite Mind.

Perfect knowledge is scientific.

10. This process is actually going forward with great rapidity as science advances. The various Physical Sciences are more and more seen as distinct, yet cognate and harmonious, divisions of one great whole. The same is true of the various branches of Psychology and Metaphysics, in their mutual coherence and interdependence: while Physics have their deepest ground in Metaphysics, in the ideas of substance and cause, without which all being is a chimera, and all science a dream. So the several sciences, physical and metaphysical, are constantly verging towards that *scientia scientiarum*, which is at once the true Philosophy and the true Ontology.

All Sciences tend to Unity.

Scientia Scientiarum: Philosophy and Ontology.

11. Philosophy and Science have been used very much interchangeably, and very much also in more or less contrast to each other. In the former case

they are used for that comprehensive view of facts and truths in the particular departments, or in the whole field of knowledge, above set forth. Thus we speak indifferently of the Science of Mind and of the Philosophy of Mind, of Natural Philosophy and Physical Science. But the words are often used with a sort of contrast, according to which science is restricted to the domain of Physics, and Philosophy is more particularly referred to Metaphysics. This is especially so when these terms are used alone, without any qualifying adjunct. Thus, if we use the word Science alone and absolutely, we usually mean Physical Science. And when we speak of Philosophy absolutely and *eminenter*, we mean Metaphysics, as including mind, which is the prime cause, and those first truths of Causality and Substance, Time and Space, which variously condition being, whether body or spirit.

Philosophy and Science further Compared and Defined.

12. As all effective thinking, or application of the laws of thought, tends, and is indispensable, to the construction of science, or thorough knowledge, so Logical Method in every department of inquiry involves the three great logical processes which mutually

Logical Method includes Definition, Division, and Reasoning.

supplement and complete each other. Definition, which unfolds the nature of the science according to its attributes or qualities: Division, which unfolds it according to its extension or the objects it includes: and Reasoning, in which we either guide our search for facts and truth, or interpret these facts by showing what can fairly be inferred from them. In regard to Definition and Division, it is unnecessary to expatiate upon them here. It is enough to refer the student to the principles already laid down on these subjects. It is only necessary to add, that exact Division and Definition are of the utmost moment to the successful investigation and treatment of any subject. We will now fix our attention on the application of the modes of reasoning to the discovery, elucidation, and proof of the truth, in regard to the object-matter so marked out by these processes. These reasonings are subject to different conditions, and have a different cogency and force, according as they are applied to NECESSARY OR CONTINGENT MATTER.

Importance of Definition and Division.

13. The former, as before defined, is that the opposite of which the mind cannot conceive without intellectual suicide. The latter is that whose existence is Con-

Necessary and Contingent Distinguished.

tingent, and the supposition of whose non-existence involves no contradiction or absurdity. These two kinds of truth give rise to the two orders of reasoning, respectively known as Demonstrative and Probable, and to the three classes of Judgments classed by logicians respectively as,

Sect. II.—Problematic, Assertory, and Apodictic Judgments.

14. The two former apply to the region of Contingent,* the last to that of Necessary truth. This distinction in judgments concerns the degree of certainty in the connection between the subject and the predicate.

A. The Problematic Judgment is neither subjectively nor objectively certain; *i. e.* it is not certain to him who holds it, nor can he enforce its acceptance upon others. It is equivalent to mere opinion.

Problematic Judgment — Opinion.

B. Assertory Judgments are true and certain subjectively but not objectively, *i. e.* sure to him who holds them, but incapable of being enforced on the acceptance of others of a

Assertory — Faith.

* This must not, however, be pressed so far as to impugn the necessary existence of God.

different moral disposition. Of this nature is belief or faith, especially Religious Faith. Its judgments are sure to the believer, although they cannot be enforced upon those of a contrary moral disposition.

Apodictic Judgments necessarily true.

C. Apodictic or Demonstrative Judgments are subjectively and objectively sure; sure to him who holds them, and capable of being enforced upon all of sane mind, who can be made to understand them and the evidence for them. Of this nature are the truths in Mathematics, Logic, and some primary axioms in Ethics and Metaphysics.

Reasoning from Apodictic Judgments sure.

15. In regard to reasoning in the sphere of necessary truth or apodictic judgments, little need be said. The conclusions of all reasoning from such judgments to others founded upon them, that conform to the principles of the syllogism in its various forms as set forth in formal logic, are as certain, and as impossible to be false, as the premises. The formal sciences afford fine illustrations of the achievements of the logical faculty in enlarging our knowledge, without in the least increasing its original materials, but by simply explicating them. The whole science of

Its power in the Formal Sciences illustrated by Geometry.

Geometry is but the logical unfolding of the contents of a few primary axioms. So also of the entire range of pure Mathematics, and of pure Logic. All necessary and *a priori* truths, intuitive and deductive, afford premises for necessary conclusions. Thus, from the *a priori* truth that space is illimitable, it follows that it is immeasurable. From the *a priori* truth, "every event must have a cause," and the minor premise, "thunder has occurred" (or been an event), it follows that this thunder must have had a cause. Here the major premise is a necessary, the minor a contingent but certainly proved truth; and the conclusion is true, with a necessity conditioned on the truth of the minor, *i. e.* in this sense, with a conditional necessity. In genuine logical reasoning the conclusion is a necessary consequence of the premises. In proportion then as they are necessarily true, the conclusion is so likewise. In this we have the type of all purely

And all Mathematics.

Reasoning from a priori truths.

From one a priori and one Contingent premise.

Sect. III.—Deductive Reasoning.

This the type of Deductive Reasoning.

16. This has place in all cases of Reasoning from wholes known in whatever way, whether of Extension or Intension, to the parts included under them; from Genus to Species, and individuals under them, or from the marks of the individual or species to the marks of those marks. So far as we have any generic truths, propositions or judgments established, whether in necessary or contingent matter, these furnish premises whence we can reason with necessary certainty, to individuals or classes contained under them. If it be established that,

(Intensive Syllogism)—"Polyps are animals,

And that

Animals have sensation,

then it follows of necessity that

Polyps have sensation."

Deductive Reasoning is from Generals to Particulars.

Deductive Reasoning then is from Generals to Particulars—the form, as we shall see, of nearly all demonstrative and absolutely conclusive reasoning.

17. But how do we obtain these universal or

Generic Judgments in Contingent Matter, when all that we know originally of mind is the individual facts that come under the purview of consciousness, and of matter, what are cognized through our senses? facts too, the opposite of which are possible, and which it is conceivable might not be repeated beyond the sphere of experience thus far had? From the fact that such persons as we have known die, how do we reach the conclusion that all men are mortal? From the fact that some water is composed of oxygen and hydrogen, how do we know that all water is so constituted? This brings us to reasoning from particular facts to a general law or truth, which is,

Whence come General Judgments in Contingent Matter?

Sect. IV.—Induction.

17. This is the principal instrument of scientific progress, and of all advance in human knowledge, except through Divine Revelation, within the realms of actual being. For this is a region of facts, objects, phenomena of actual existence which are first known as individuals, and might or might not be, according to the good pleasure of God.

By Induction, or reasoning from Individuals to Genera.

Actual beings are Individuals.

Scope of Formal Sciences.

The Formal Sciences and Metaphysics do not of themselves discover or prove any actual being. They only show certain necessary conditions or consequences of any facts of actual being, which may be brought to light by the other cognitive powers.

All progress in Scientific Knowledge of actual being is from Individuals to Classes.

But all advance in the knowledge, and especially the scientific knowledge of actual being, is by ascent from particular facts to general laws. It proceeds therefore, from what we know in some cases, to infer that the like is true in all similar cases. This is induction or inductive generalization.

Induction more than Generalization.

Induction, however, is more than Generalization, which it always includes. There may be generalization without induction, though there can be no induction without generalization. Generalization combines in a class objects having similar qualities, and denotes them by a class-name. Induction concludes from the fact that some of a given class already generalized possess some given property, all others of that class possess it—in other words, that because a certain mark A is, in some cases, attended with a certain mark B, it is so in all other cases.

In what respect it is so.

Thus, from the fact that some fire tortures living flesh in contact with it, we reason inductively that all fire will do it. Here is generalization in this way, and to this extent, that what is found true of some, is extended to all, that have a given mark.

Example.

18. The great question then, in regard to this class of cases, which needs to be determined, is, when are we warranted in taking some instances that have come under our knowledge, as samples or accurate representatives of a whole class, including, it may be, like cases innumerable? What are the criteria which distinguish these crucial instances from others which warrant no such inference?

The great question regarding Induction.

What are tests or crucial instances.

19. There is the test of a complete enumeration of all the instances or individual cases composing the class in question. If these all, without exception, have the property in question, then it of course belongs to the whole class. Thus, if the season of greatest growth is in May, June, July, August, which are the only months whose names are without the letter *r*, then the general conclusion follows

Simple Enumeration.

Example.

with absolute certainty, that the months without the letter *r* are those of greatest growth. If it has been found from actual observation, that each of the planets moves in an elliptical orbit, then it is true beyond a peradventure, that all the planets move in such orbits. This is what Bacon called Induction *per simplicem enumerationem*, by the mere enumeration of all the cases involved. It has also been named Empirical Induction, because its compass is limited to actual experience, and it detects no cause and establishes no law reaching beyond such experience. It is therefore comparatively unimportant. That induction alone is fruitful which enables us to go beyond such cases as have fallen within our experience, to an indefinite number of like cases, *i. e.* all of the same class not yet brought within the range of our experience.

Why this is Empirical Induction.

And unimportant.

The only fruitful Induction.

20. In order to this, it is necessary to ascertain, not only the empirical fact, that, in such instances as have fallen under our cognizance, the phenomenon in question has occurred, but that there is a causal agency, or other uniform concomitant, connected with them

Requisites to it.

A Causal Agency.

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which ensures it, and which attends all like instances. Then, when it is settled what is the cause or mark of any given phenomenon, the principle that like causes produce like effects, which is either self-evident, or so nearly so that all mankind act upon it, induces the conclusion that, in all similar cases, we may anticipate a like phenomenon.

21. The difference between such an induction and that which is purely empirical, *per simplicem enumerationem*, is strikingly illustrated in the second example of the latter kind, above given, which was the inductive conclusion that all the planets move in elliptical orbits, from observing that each of them moves in such orbits. This, however, of itself, creates only a moderate presumption, that any planets now unknown and yet to be discovered, move in such orbits. But when it was ascertained that the Centripetal and Centrifugal Forces act jointly on all the planets, and that the product of this joint action is an elliptical orbit, then the conclusion was indisputable, that all planets observed and unobserved, move in elliptical orbits.

Difference from Simple Enumeration Illustrated.

Causal Force Discovered.

22. What then are the Tests of such Causal

Agency, or other equivalent concomitant, and proof of a given phenomenon? The proofs that a given object or agency is, or contains in itself, the cause, or invariable concomitant of a given effect, so that we are warranted in asserting that the instances observed are as good as the entire class of like instances? The Inductive Syllogism naturally falls into the Third Figure. Thus:

Tests of Causal Agency, etc

Inductive Syllogism.

"X Y Z have polarity,
X Y Z are (represent *quoad hoc*) all magnets,
∴ All magnets have polarity."

It may, however, be put more awkwardly, in the First Figure. Thus:

"X Y Z have polarity,
All magnets are (represented *quoad hoc* by) X Y Z.
∴ All magnets have polarity."

In either case the question is, when do the particular cases X Y Z so represent the whole class, or when are they so proved to be, or to contain the causes or uniform concomitants of a given phenomenon, that they fairly represent the whole class, and warrant a universal inductive conclusion?

The Question regarding it.

1st CRITERION, THE METHOD OF AGREEMENT. —If, whenever a given object or agency is present, without counteracting forces, a given effect is produced, there is strong evidence that we have found the true cause of the effect, which will always produce it, in the absence of counteracting forces. Thus, if, in all cases of the application of given degrees of heat, clay hardens, lead melts, and water boils, it is just to conclude that this is the real cause of these phenomena, and that whenever it is applied in such measure to these several substances, they will re-occur. It is to be borne in mind, however, that the same effect may proceed from different causes. In order to determine to which of two possible causes it is due, in any given cases, the distinctive indications of each respectively must be sought. This is usually not difficult. The sensation of heat may arise from the general warmth of the weather, from an artificial fire, from excessive clothing, or from fever. It is usually easy, in view of all the circumstances, to determine which. But if not, unreal causes may be eliminated by the

1st Test is the Method of Agreement.

Example.

Exception. Same effect may proceed from different causes.

2d CRITERION; THE METHOD OF DIFFERENCE.

—This is given when, the supposed cause being present the effect is present, and this being absent the effect is wanting, *i. e.* unless in the latter case other counter-agents are present to neutralize it, or in the former to produce it. Thus, it is double proof that sound is the result of vibrations of air excited by the resonant body, if, on the one hand, whenever sound is heard, such vibrations are found; whenever such vibrations appear sound is given forth; and if, on the other hand, a bell or other sonorous body, suspended and struck in an exhausted receiver, yields no sound. It proves that the contact of moisture is the cause of the decomposition of animal matter, if, whenever the latter occurs such moisture is present; if dryness checks or arrests it; and if salt, which prevents it, acts by detaching the water from the meats which it preserves. If, when reason is present, there is accountability, and when it is absent there is none, then it is a condition of accountability.

2d Test. Method of difference.

Exception.

Example.

3d CRITERION—accounting for residual variations without invalidating the proof of the supposed cause. Thus, it was found that sound traveled faster than what

3d Test. Residual variations accounted for.

seemed the true theory of its law of velocity allowed. It was suspected, however, that the rarefaction of the air, arising from the heat produced by the motion of the sound, accelerated its progress to this extent. Experiments proved this conjecture true, and thus confirmed the original hypothesis.*

* The following striking example is given in the words of *Thomson's Laws of Thought*, New York Edition, pp. 262-3, Chap. VII.

"In Sir Humphrey Davy's experiments upon the decomposition of water by galvanism, it was found that besides the two components of water, oxygen and hydrogen, an acid and an alkali were developed at the two opposite poles of the machine. As the theory of the analysis of water did not give reason to expect these products, they were a *residual phenomenon*, the cause of which was still to be found. Some chemists thought that electricity had the power of *producing* these substances of itself; and if their erroneous conjecture had been adopted, succeeding researches would have gone upon a false scent, considering galvanic electricity as a *producing* rather than a *decomposing* force. The happier insight of Davy conjectured that there might be some hidden cause of this portion of the effect; the glass vessel containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it, so that the water would have no share in their production. Assuming this he proceeded to try whether the total removal of the cause would destroy the effect, or at least the diminution of it cause a corresponding change in the amount of effect produced. By the substitution of gold vessels for the glass without any change in the effect, he at once deter-

4th Criterion. Concomitant Variations.—

4th Test. Concomitant Variations.

If, as the amount of the supposed cause varies, the effect varies proportionally, it is strong evidence of its being the real cause. "That the column of mercury in the Torricellian tube was counterpoised by a column of air, was proved by Pascal when he caused the instrument to be carried up the mountain, and found that as the ascent gradually diminished the height of the column of air above it, so was the column of mercury it was able to sustain diminished in proportion."

mined that the glass was not the cause. Employing distilled water he found a marked diminution of the quantity of acid and alkali evolved; still there was enough to show that the cause, whatever it was, was still in operation. Impurity of the water then was not the sole, but a concurrent cause. He now conceived that the perspiration from the hands touching the instruments might affect the case, as it would contain common salt, and an acid and an alkali would result from its decomposition under the agency of electricity. By carefully avoiding such contact, he reduced the quantity of the products still further, until no more than slight traces of them were perceptible. What remained of the effect might be traceable to impurities of the atmosphere, decomposed by contact with the electrical apparatus. An experiment determined this; the machine was placed under an exhausted receiver, and when thus secured from atmospheric influence, it no longer evolved the acid and the alkali."

When either of these criteria is found, free from conflicting evidence, and especially when several of them concur, the evidence is clear that the cases observed, are fair representatives of the whole class, and warrant a valid universal inductive conclusion.

Sect. V.—Hypothesis.

23. But why make observations and experiments in one direction, or for the purpose of testing one view of the cause of given phenomena, rather than any other? It can only be because the mind entertains some conjecture or suspicion that this may correspond with the facts. Thus it is led to institute investigations and trials for the purpose of testing the truth of this conjecture. Such a conjecture so entertained is a Scientific Hypothesis, which is thus but a provisional and tentative theory, while a true theory is a proved hypothesis. Such hypotheses, although they have often been abused, by the premature or unwarrantable assumption of their truth, are indispensable to effective progress in science. Without such a guide and stimulus, all observations and experiments would be aimless, and com-

Reason for Hypothesis.

Hypothesis, Reasonable Conjecture or Tentative Theory.

Use and Necessity of Hypothesis.

monly fruitless. Indeed, for the most part, they would be unattempted. Investigations so guided have led to nearly all the great achievements of scientific progress.

How far Theory and Hypothesis are synonymous.

24. Some confound Theory with Hypothesis, and accurate writers often find it difficult to use them so as to avoid all shades of synonymous meaning. But nevertheless, correct use points towards the difference we have indicated. Hypothesis could not be well substituted for Theory, when we speak of Wells' theory of dew, or Dalton's theory of definite chemical proportions, or the Newtonian theory of universal gravitation. And yet theory is often used for hypothesis, *i. e.* for an unproved doctrine or speculation, or a tentative and provisional, but uncertain explanation of phenomena. Thus we speak of Smith's Theory of the Moral Sentiments; the exploded phlogiston and anti-phlogiston theories. Some use theory for a provisional and unproved explanation of a large group of facts. This however is but an hypothesis regarding such a group of facts.

Definition of some Scientific Terms.

Judgments.

In regard to the distinction between Theoretical and Practical Judgments, and other Scientific Terms, we

quote the following from Thomson's *Laws of Thought:*

"Judgments that relate to speculation only, are called Theoretical; those which refer to practice are Practical. Judgments that require or admit of proof, are called Demonstrable; those which are manifest from the very terms, are Indemonstrable. Thus much being premised we can define certain subordinate parts of a science.

Judgments Theoretical, Practical, Demonstrable, Indemonstrable.

An Axiom is an indemonstrable theoretical judgment. A Postulate is an indemonstrable practical judgment. A Theorem is a demonstrable theoretical judgment. A Problem is a demonstrable practical judgment. A Thesis is a judgment proposed for discussion and proof (but with Aristotle it sometimes means an axiom of some special science or disputation). An Hypothesis is a judgment provisionally accepted as an explanation of some group of facts, and is liable to be discarded if it is found inconsistent with them. A judgment which follows immediately from another, is sometimes called a Corollary or Consectary. One which does not

Axiom, Postulate, Theorem.

Problem.

Thesis.

Hypothesis.

Corollary.

properly belong to the science in which it appears, but is taken from another, is called a Lemma. One which illustrates the science where it appears, but is not an integral part of it is a Scholion."

Lemma.

Scholion.

25. The great distinction of Scientific Genius lies chiefly in this insight which, with keen discernment of analogies, anticipates the truths or laws of nature, and devises observations and experiments to prove or disprove them. So Newton suspected that the same force which causes the falling of an apple, propels all matter, and produces the revolution of the planets; Franklin, that lightning is a discharge of electricity. They proceeded to verify these hypotheses by experiments and observations which proved them. While the legitimate use of hypothesis is thus advantageous and essential to science, the cautions needful to be observed to prevent the abuse of it are,

Chief mark of Scientific genius.

Cautions in Regard to Hypothesis. 1. Must be needed.

A. No hypothesis should be assumed to account for what can be otherwise accounted for, on existing and known principles.

2. Adequate.

B. It should be adequate to account for the phenomena in question.

C. The facts to be accounted for should be real and not imaginary, as the question before mentioned of Charles II. to the Royal Society, why a live fish in water would increase its weight, while a dead fish would not, and quite perplexed some of its members, until it occurred to them to inquire if the fact were so.

3. The facts to be explained must be real.

D. It should be independent of subsidiary hypotheses — it should not require other hypotheses to account for itself.

No Subsidiary Hypotheses.

E. It should not be assumed to be true until proved to be so.

To be accepted when proved.

Sect. VI.—Analogy.

26. When it is argued from a known resemblance between objects or classes in some known particulars, that they resemble each other in other respects, this is reasoning from analogy. It has been common to define analogy as a proportion between objects. When we reason that because men resemble animals in having life and sensation, they therefore resemble them in the power of locomotion, or in the grade of their intelligence, we reason from

Reasoning from Analogy defined.

Example.

analogy, or the relative proportion of objects.* It is obvious that this is a very uncertain argument, and can, in no case, rise higher than mere probability. This probability will be weaker or stronger according to circumstances. The argument for future retribution, from the present evils visited upon sin, is certainly stronger than the argument that brutes have reason because other conscious beings have it. But in neither case is it conclusive. The argument from analogy may be well employed to add a cumulative force to other arguments. It is not, however, in any case conclusive of itself.

Has only a probable force.

May strengthen other arguments.

27. Its most important service, however, is in refutation of fallacious arguments. It often has in this way a powerful negative force. Thus, if it be objected to the doctrine of future punishment that the infliction of pain is inconsistent with the benevolence of God, this argument is refuted by the fact

Most useful in refutation.

Examples.

* To reason from Analogy, is to reason from the Intension of that to which it relates. To reason by Induction is to reason in extension from one or some objects in a class to all in that class. In analogical reasoning, we argue from a resemblance in some qualities to a resemblance in other qualities.

that God does inflict pain, or so order and permit events that it is undeniably inflicted, in this life. The alleged impossibility of the future life and immortality of the body on account of its death, is disproved by the fact that in all nature life is evolved from death, and the seed which we sow "is not quickened except it die." 1 Cor. xv. 36.

Sect. VII.—Categories.

28. These are summa genera of predicables. Logicians and metaphysicians have sought to give complete lists of these *summa genera*, to which all particular predicables and classes of predicables might be referred. It has, however, been hard to find any such exhaustive enumeration. Says Whateley, "The Categories enumerated by Aristotle, are *οὐσία, πόσον, ποῖον, πρός τι, ποῦ, πότε, κεῖσθαι, ἔχειν, ποιεῖν, πάσχειν*; which are usually rendered, as adequately as, perhaps, they can be in our language, substance, quantity, quality, relation, place, time, situation, possession, action, suffering. The catalogue (which certainly is but a very crude one) has been by some writers enlarged, as it is evident may easily be done by subdividing some of the

Definition of Categories.

Aristotle's Categories.

heads; and by others curtailed, as it is no less evident that all may ultimately be referred to the two heads of *substance*, and *attribute*, or (in the language of some logicians) *accident*." Some, however, perhaps justly, translate ἔχειν, "mode of action," instead of "possession." Aristotle's Categories are rather metaphysical than logical.

Kant's Categories.

29. Kant's celebrated four triplets of Categories are certainly ingenious, and, if not absolutely exhaustive, in a metaphysical view, go far to show the nature and *a priori* basis of the several logical judgments. According to him all judgments must connect the predicate with the subject so as to involve under the head of,

1. *Quantity.*	2. *Quality.*	3. *Relation.*	4. *Modality.*
Unity,	Affirmation,	Substance and Accident,	Possibility,
Plurality,	Negation,	Cause and Effect,	Reality,
Totality.	Limitation.	Action and Reaction.	Necessity.

It may be observed that the first of these triplets corresponds to Singular, Particular, and Universal Judgments; the second to Affirmative, Negative, and Restrictive* Judgments; the third to Categori-

* Restrictive Judgments "are such as contain a negative in the predicate-conception, *e. g.*, God is infinite. The human soul is immortal. In respect to their contents, they are negative; but

cal, Conditional, and Disjunctive Judgments; the fourth to Problematic, Assertory, and Apodictic Judgments.

30. Tables of Categories are almost as various as the writers on Logic and Metaphysics. McCosh gives the following as a provisional summary of primary judgments. By McCosh.

1. Identity and Difference.
2. Whole and Parts.
3. Space.
4. Time.
5. Quantity.
6. Resemblance.
7. Active Property.
8. Cause and Effect.

31. J. S. Mill in his Logic gives the following classification of nameable things in the spirit of the Positive Philosophy. J. S. Mill.

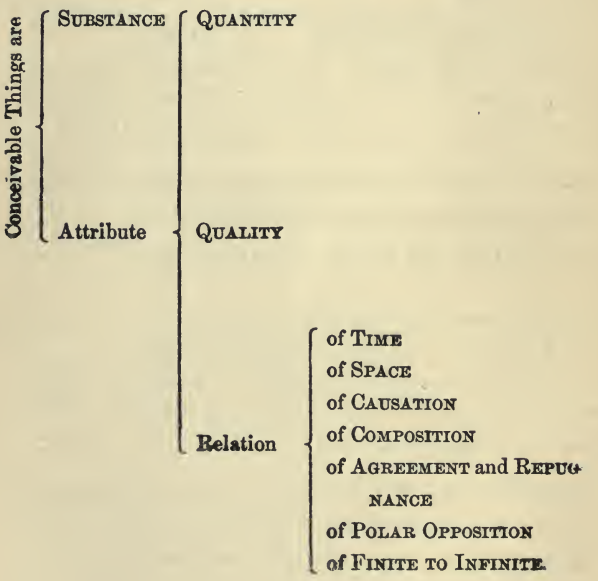
1. Feelings or states of consciousness.
2. The minds which experience these feelings.
3. The bodies or external objects which excite certain of these feelings, together with the power or properties whereby they excite them.

In respect to form, they are affirmative. Logically considered, therefore, they belong to the class of affirmative judgments. These judgments are also called *infinite*, or more properly *indefinite*, because, by means of a predicate involving a negative, the subject is transferred from the sphere of definite conception to that of indefinite conception, a sphere to which it does not properly belong."—*Gerhart's Philosophy and Logic*, p. 214.

4. The successions and coexistences, the likenesses and unlikenesses between feelings and states of consciousness."—*Logic*, I. 111.

32. Thomson (*Laws of Thought*, p. 315) just attempts the following:

TABLE OF THE CATEGORIES.



SECT. VIII.—HARMONY AND CO-ORDINATION OF SCIENCES.

33. As the application of scientific method to any given and mutually related set of phenomena or truths develops a science of these facts, like the Science of Botany, Anatomy, Ethics, etc., so many of these sciences are related to each other as Genus and Species. Thus Ornithology, Piscatology, etc., under Zoology. Various attempts have been made to classify the Sciences so as to show their Mutual Harmony and Interdependence. It is plain that they might be logically divided and sub-divided from various stand-points, which have been taken according to the respective aims and purposes of the authors. Thus they may be divided into the Speculative and Practical, or the Physical and Metaphysical, or the Formal and Material, etc., with their respective subdivisions. Attempts of this sort have often been made, with considerable success and utility.

Classification and Mutual Harmony of the Sciences.

Speculative and Practical, etc.

34. Compte and the positive school of philosophers, however, amidst their enormous errors, have unfolded a scheme of classification and co-ordination among the sciences, at once beautiful and fruitful,

which has commanded wide acceptance among those who have attended to the subject.

Starting with Descartes' suggestion, that the order of arranging the sciences should be from the simplest to the more complex, he adopts the following, which at once commends itself by its simplicity, naturalness, and beauty, and which we give, as we find it, in a form most available for our present purpose, in *Thomson's Laws of Thought*, pp. 316–19.

"Mathematics, or the science of quantity, is at once the most simple in its elements and the most general in its application, entering more or less into all the sciences of nature, and constituting almost the whole of that which comes next it in the order of dependence. Astronomy, or the science of the heavenly bodies, is the application of mathematical truths to the laws of matter and motion; matter and the motions of material bodies being the new conception which belong to this science. Physics, being the science, or rather group of sciences, which is conversant with the general laws of the world, so far as they relate to beings without life or organization, would come next; and it imports, in addition to the conceptions of Astronomy, those of light, of

heat, of sound, of electricity, of magnetism, and many others. Chemistry would rank next, which is the science of the decomposition and combinations of the various substances that compose and surround the earth. Next in order of complexity would rank Physiology, founded on the additional conception of vegetable and animal life. To this would succeed Anthropology, or the science of man's nature; and to this Social Science, which ascertains the laws that govern men when combined in cities and nations. Each of these departments may be divided into many branches; as Physics into Acoustics, Optics, Electricity, and the like; or Social Science into Morals, Politics, Political Economy, Law, and the like.

"On comparing scientific works, differences in the mode of teaching the same subject become apparent. In one the pure theory of Astronomy is presented; in another the striking features of its historical progress as a science, with speculations on the historical sequence of the phenomena themselves; in a third the practical applications of which the science admits in respect to the comfort and progress of mankind. This threefold mode of treatment runs through all the sciences; and in a table of

them might well be expressed. The classification would thus embody all that is valuable of another system of classes, that according to the purpose towards which the science was directed.

"A classification which advances on Descartes' principle, from the more simple to the more complex subjects, which commences from the notions of extension and quantity, and proceeds through material things, up to living, intelligent, and moral agents, ought to coincide with the order in which the sciences themselves have reached maturity. And this it certainly does. Mathematics had made good its ground when astronomy was yet in its infancy; physics began to obtain a sure footing later than either; whilst the sciences which relate to life are still very immature; and some of the main problems of social science are yet matter of controversy even in our own days.

"There is besides a general correspondence between this classification and the order in which the various objects of science came into being. The heavenly bodies were first appointed their paths in the celestial spaces; then the surface of our earth was prepared for living creatures; then they were created after their kind, and man the last. The social life

of man grew up last of all, when his race was multiplied on the globe; and ever as new elements appear, the conditions of society are being modified even to the present time."

Hence emerges the following

"CLASSIFICATION OF THE SCIENCES.

Group.	*Mode of Treatment.*		
I. MATHEMATICS	Theoretical.	Historical.	Applied.
II. ASTRONOMY	Theoretical.	Historical.	Applied.
III. PHYSICS	Theoretical.	Historical.	Applied.
IV. CHEMISTRY	Theoretical.	Historical.	Applied.
V. PHYSIOLOGY	Theoretical.	Historical.	Applied.
VI. ANTHROPOLOGY	Theoretical.	Historical.	Applied.
VII. SOCIAL SCIENCE	Theoretical.	Historical.	Applied.

RELIGIOUS PHILOSOPHY."

APPENDIX.

APPENDIX A.

EXAMPLES FOR PRAXIS.

☞ *The following examples may be used for practical exercise in Conceptions, Judgments, and Reasonings of all kinds. In analyzing Syllogisms, let the student complete them when unfinished, and point out their kind, whether Categorical or Hypothetical; if the former, give their Mood and Figure; if the latter, show whether they are Conditionals, Disjunctives, or Dilemmas. Mark the Enthymemes, Sorites, Prosyllogisms, and Episyllogisms. In all cases show whether the Syllogism is valid or invalid, and if invalid, indicate the kind of Fallacy.*

1. Body is extended substance,
   This inkstand is a body,
   ∴ It is extended substance.

2. Plants are bodies with organization,
   Potatoes are plants.
   ∴ .....

3. Animals are bodies having organization and sensation,
   Frogs have organization and sensation.

∴ .....

4. Bodies having organization, sensibility, and reason are men,
   The poets are men.

∴ .....

5. X Y Z, are ruminant,
   X Y Z, are (as good as) all horned cattle.

∴ .....

6. Quadrupeds are animals,
   Worms are animals.

∴ .....

7. Oaks are vegetable,
   Oysters are not oaks.

∴ .....

8. Beasts are animals,
   Birds are not beasts.

∴ .....

9. These emigrants are either Scotch, Irish, or German,
   They are not Germans.

∴ .....

10. These people are patriots because they are free.

11. If the classics teach how to produce wealth they ought to be studied,
    They do not so teach.

∴ .....

12. If we can prevent what occurs we ought not to fret about it,
    If we cannot prevent what occurs we ought not to fret about it,
    But either we can or cannot prevent it.
    ∴ .....

13. A Christian nation is brave,
    A brave nation is free,
    A free nation is happy.
    ∴ .....

14. A plane triangle is a rectilineal figure having three sides,
    A plane triangle is A B C.
    ∴ .....

15. All these trees make a thick shade,
    This catalpa is one of these trees,
    ∴ It makes a thick shade.

16. Whatever study gives knowledge relative to either of the three learned professions ought to be a part of liberal education;
    Geology and Mathematics do not give such knowledge,
    ∴ They ought not to be studied.

17. If all men are liars then nothing can be proved by human testimony;
    But some things can be proved by human testimony,
    ∴ No men are liars.

18. Typhoid fever is epidemic,
    Because A. B. and C. have it.

19. An inflated currency promotes national prosperity, because it enables persons to make rapid fortunes.

20. What we eat grows in the fields or is the flesh of animals,
Cooked food is what we eat,
∴ Cooked food grows in the fields or is the flesh of animals.

21. The rumor that A. B. has committed a given crime is universal, for I heard it from Mr. A. and Mr. B.

22. If we say the Baptism of John was from heaven we condemn ourselves for not believing him;
If we say it was of men, the people will stone us;
But we must, if we say any thing, confess it was from heaven or of men;
∴ If we say any thing, we must either condemn ourselves, or the people will stone us. Luke xx. 4–6.

23. Some flowers are (all the) tulips,
All flowers are beautiful,
∴ All the tulips are beautiful.

24. All false religions have sustained their claims by alleged miracles,
Christianity sustains its claims by alleged miracles;
∴ It is a false religion.

25. The minute-hand can never overtake the hour-hand of a clock, because while it is passing to the point where the hour-hand is at any given moment, the latter will have advanced

some distance: and when the former has passed over this distance the hour-hand will have advanced still further; and so on ad infinitum.

26. This man has an excellent character because he belongs to an excellent church, as appears from its being composed of such excellent men.

27. He who is most hungry eats most,
He who eats least is most hungry,
∴ He who eats least eats most.

28. If the taking of an oath to discharge our duty tends to secure its performance, then it ought to be repeated in reference to every duty of life; if it does not, then the civil oaths administered are superfluous. But one or the other of these is true. ∴ The oaths commonly administered are superfluous, or they should be repeated in connection with every duty of life.

29. No man is rich who is not content,
No miser is content (*i. e.* every miser is one who is not content),
∴ No miser is rich.

30. Men can live without animal food, and they can live without vegetable food, as has been often demonstrated,
But all food is either animal or vegetable,
∴ Men can live without food.

31. He who calls you a man speaks truly,
He who calls you a fool calls you a man,
∴ He who calls you a fool speaks truly.

32. Useful studies ought to be encouraged,
Logic, since it helps us to reason accurately, is such,
∴ It ought to be encouraged.

33. X Y Z have polarity,
X Y Z are (as good as) all magnets,
for polarity appears wherever magnets are; it disappears when they are withdrawn, unless other polar forces are present, and it increases with the power of the magnet;
∴ All magnets have polarity.

34. Some men of genius are (all) the poets,
Some poets are melancholy.
∴ .....

35. The mind is a thinking substance,
A thinking substance is a spirit,
A spirit has no composition of parts,
That which has no composition of parts is indissoluble,
That which is indissoluble is immortal,
Therefore the mind is immortal.

36. Protagoras engaged to teach Euathlus the art of pleading for a large reward, one half to be paid at once, the other half when the latter should have gained his first cause in court. After a short time Protagoras sued Euathlus for the unpaid moiety, enforcing his claim by the following Dilemma:

If the case is decided in my favor, the sum will be due to me according to the finding of the court;
If it is decided in your favor, the sum will be due to me according to our contract,

But it must be decided either in my favor or yours.

∴ Whether I gain or lose the cause I shall be entitled to the reward.*

Euathlus thus answered:

If I gain the cause, nothing will be due you according to the decision of the court,

If I lose it nothing will be due you according to our contract;

But I shall either gain or lose it,

∴ In neither case shall I pay you the reward.

37. A policy which promotes the national wealth ought to be adopted;

But the education of the people increases their wants and expenditures, and therefore does not increase national wealth;

∴ It ought not to be adopted.

38. All is not gold that glitters,

Tinsel glitters,

∴ It is not gold.

39. If there had been a law that could have given life, then verily righteousness should have come by the law,

But righteousness did not come by the law."

∴ ..... Gal. iii. 21.

* The fallacy here is that the Disjunction is incomplete. There is another horn, viz: that Protagoras had no cause of action, because before the bringing of this suit, Euathlus had no case in court. See Chap. VI., 28.

40. Poets are men,
Orators are men.
∴ .....

41. Plants are bodies with life, and without consciousness,
Geraniums are such bodies.
∴ .....

42. All trees bearing acorns are oaks,
Some trees do not bear acorns.
∴ .....

43. All men are rational animals,
Apes are not men,
∴ They are not rational animals.

44. Some men are orators,
Some bipeds are (all) men,
∴ Some bipeds are orators.

45. The following answer was given to Pyrrhus' assertion that nothing can be certainly known:

If you certainly know this, your assertion is disproved,
If you do not certainly know it, you have no right to affirm it,
But you either do or do not know it,
Therefore your doctrine is untenable.

46. Most people are careless,
Most people are destitute of perfect health.
∴ .....

47. It is almost certain that C. D. is a true witness because there is a probability amounting to $\frac{2}{3}$ that he saw and ob-

served correctly what he testifies about, and another probability of $\frac{2}{3}$ that he would tell the truth if he did know it.

48. What is the probability that B. B. wrote a certain anonymous letter, where the separate probabilities are—

From chirography, $\frac{1}{2}$,
From the sentiments, $\frac{1}{2}$, and
From his known meanness of character, $\frac{1}{4}$?

49. All plants contain cellular tissue,
No animals are plants,
$\therefore$ No animals have cellular tissue.

50. A fungus is a plant, for it is either a plant or an animal, and it is not an animal.

51. If this man were wise, he would not speak irreverently of Scripture in jest; and if he were good, he would not do so in earnest; but he does it, either in jest or earnest;

$\therefore$ .....

52. Magnets attract iron, and they have polarity,

$\therefore$ .....

53. Insects are not winged, because birds are winged, and insects are not birds.

54. Insects are not fishes, because fish live in water, and insects do not.

55. What is the probability of a recurrence next year of a fire in one of our great cities, consuming millions of property, from the fact that such fires have occurred on an average once a year for the last ten years?

56. All these students form a powerful body,
A. B. is one of these students,
He forms a powerful body.

57. What inferences by opposition can be obtained from—
All men are fallen,
Some trees are oaks,
Some stones are not diamonds,
All triangles are three-sided figures?

58. Convert logically—
Some men are not cultivated.
Some men are cultivated.
All plants are living things.
All pure men love God.

59. What do you say of dividing college studies into chemical, classical, rhetorical, theoretical, practical, those taught by text-book, and those taught by lecture, as a sample of Logical Division?

60. What do you say of defining a square as a rectangular parallelogram, a quadruped as a dog, a triangle as a three-sided figure whose angles are equal to two right angles?

61. Point out the subject, predicate, copula, quanity, and quality of the following judgment:
All students decline in scholarship who become indolent.

62. Body is space-filling substance,
Body has mobility,
∴ Whatever has mobility is space-filling.

63. Discontented persons are not wise,
Abstemious persons are wise,
∴ Discontented persons are not abstemious.

64. Whatever promotes the public good should receive the attention of government,
Dram-shops do not promote the public good,
∴ They should not receive the attention of government.

65. Animals are sensitive beings,
Sensitive beings have life,
Beings having life are organized,
∴ Animals are organized.

66. Useful pursuits ought to be encouraged;
Farming, since it produces food for man and beast, is such;
∴ It ought to be encouraged, and consequently what depresses it ought to be opposed.

67. If the earth does not revolve on its axes, existing prenomena cannot be accounted for;
It does so revolve,
∴ .....

68. Silver is money,
Gold is money,
∴ .....

69. Perseverance enables us to conquer,
Therefore it is useful.

70. All gold is precious,
This mineral is precious,
∴ .....

71. Warm countries alone produce oranges,
Florida is a warm country,
∴ Florida produces oranges.

72. The earth moves either in a circle, ellipse, or straight line,
It does not move in a straight line,
∴ .....

73. If the atheists are right the world exists without a cause,
But the atheists are not right,
∴ .....

74. A religion attended by miracles is from God, for none but God can work miracles, and he would not work them in behalf of an impostor;

The Christian religion was attested by miracles, for indubitable evidence proves it;

∴ The Christian religion is from God.

75. Point out the subject, predicate, copula, and distributed terms in the following judgments:

He that believeth hath eternal life.

Civil office is either legislative, executive, or judicial.

Circles are figures equidistant at every point from a point within.

76. What can you infer by Opposition and Conversion from—

All men are liable to become gray,
Some stars are (all the) planets,
Some persons are not cheerful?

77. Although animals feel, perceive, remember, and have instinct like men, they do not know axioms above sense, nor have articulate speech, nor increase their knowledge from gen-

eration to generation, which are higher gifts than the former. Men do this, and therefore have more than brute intelligence.

78. All plants have life,
But none have consciousness,
∴ .....

79. If I am to succeed in life, no conduct of mine will prevent it;
If I am to fail, no conduct of mine will prevent it;
But one or the other of these is true;
∴ All effort on my part is useless.

80. The predictions of weather from Washington having been true four days out of five, and it being proposed to increase their accuracy—one-tenth by better instruments, one-tenth by increased skill in the observers, and one-tenth by the increase of stations of observation—what measure of probability may we expect in the predictions of next year?

81. Many defalcations and breaches of trust have recently been discovered, and therefore nobody is worthy of confidence.

82. What can you infer by all modes of Logical Inference, mediate and immediate, from the following judgments, taken singly or in any mode of combination? State and analyze also the kind of reasoning, or of syllogism, employed in each instance:

A., B., and C. are scholars,
All scholars are studious,
All studious persons are diligent.

83. 1. The following definitions of Logic have been given. Test each of them by the proper logical canon for good definition:

a. Logic is the science of reasoning.

b. Logic is the art of reasoning.

c. Logic is the science of thought.

d. Logic is the science of discursive thought.

e. Logic is the science of the laws of discursive thought.

f. Logic is the art of improving the mind.

2. Give a logical division of conceptions according to their quality, as clear, distinct, etc. (See Chap. II., 9.)

3. What can you conclude by any form of inference, mediate or immediate, from—

Men are capable of religion,

Apes are not capable of religion?

84. "The gospel which was preached by me is not after man, for I neither received it, neither was I taught it [by man]; but by revelation of Jesus Christ." Gal. i. 11, 12.

85. "I do not seek to please men, for if I did, I should not be the servant of Christ [as I am]." Gal. i. 10.

86. "But that no man is justified by the law in the sight of God is evident: for, the just shall live by faith, and the law is not of faith." Gal. iii. 11, 12.

87. The government cannot outlive the prevalence of Christianity among the people, because it is a republic, and republics are governed by the people, and only virtuous people are fit to govern; but all experience proves that a people will not remain virtuous after it has rejected Christianity.

88. Paganism, Mohammedanism, Judaism are false;

These (according to infidels) are representative of all religions;

∴ .....

89. Reason, conscience, and free-will are requisite to accountability, because all possessing them, whether angels or men, are accountable; while those destitute of either of them, as infants, idiots, and maniacs, are not accountable.

90. What can you infer from each of the following judgments, singly, or from any combination of some or all of them?—

All men are rational creatures,
Rational creatures are progressive,
Brutes are not progressive.

91. The ice-crop has failed in a certain region three times in twenty years. Compute the probability of its not failing the present winter.

92. Analyze the two following, and point out their different logical force, with reasons:

1. Parallelograms are plane figures,
Circles are not parallelograms,

∴ .....

2. Parallelograms are four-sided plane figures whose opposite sides are parallel,
Circles are not parallelograms,

∴ .....

93. Men are bipeds, so also are birds, and therefore birds are men.

94. Men of genius are eminent poets, and
Men of genius are eminent philosophers,

∴ .....

95. All adequate cognitions are distinct,
All distinct cognitions are clear,
No clear cognitions are obscure.

What can you infer from each of the foregoing?—

1. By Opposition;
2. By Conversion;
3. By the combination of any two of them;
4. By the combination of all of them.

96. The cases of bad drainage known are cases attended with much sickness,

These cases are (as good as) all cases of it,

∴ .....

97. Bad weather is followed by good weather, and good weather by bad weather,

∴ The one is the cause of the other.

98. Supposing the prospects of the completion of a proposed building in six months to be nine-tenths from the known promptitude of the architect and builder, three-fourths from the possibilities of rapid work in winter, and three-fourths from various other contingencies, what is the total chance?

99. If it is our destiny to be saved, we shall be saved whatever we do or omit to do; and if we are to be lost, we shall be, whatever we may do about it. But we are destined to be saved or lost; whatever we do, therefore, will not affect our destiny.

100. Moisture is a chief cause of animal putrefaction, because—

a. It is present where such putrefaction occurs.

b. Putrefaction is arrested in a perfectly dry atmosphere.

c. Any variations from this are accounted for by the action of other agents.

101. What do you say of the following as specimens of Definition?—

a. The United States are composed of the territory and its inhabitants, whose central government has its seat at Washington.

b. The United States are composed of the New England, Middle, Southern, South-western, North-western, and Pacific States.

102. Give your opinion of the following definitions and divisions of, or under, Applied Logic:

a. A Fallacy is an unsound mode of reasoning. Logical Method is the true mode of reasoning.

b. Fallacies are divisible into Formal and Material, illicit process, *argumentum ad verecundiam*, and *post hoc ergo propter hoc.*

103. What can you infer from the first of the following judgments by Conversion; from the second by Equipollence or Infinitation; from the third by Opposition; from any combination of them by Mediate Inference?—

The precious metals are gold and silver,
Gold and silver are fusible,
Fusible metals are evenly divisible.

104. What can you infer by any mode of Mediate or Immediate Inference from the following judgments, whether taken singly or in any possible combination of some or all of them?—

Some animals are birds,
All birds are winged creatures,
No winged creatures are reptiles.

105. The Bible is a revelation from God if it is attested by adequate miracles,

It is a revelation from God,

∴ .....

106. Having for a major premise,

"Money is either gold, silver, nickel, copper, or paper," what can you infer from it with either of the following as a minor premise:

a. This money is gold.

b. This money is not copper.

107. If wars enrich many people, they promote the public welfare;

If they impoverish many people, they are hostile to the public welfare;

But they both enrich many people and impoverish many people,

∴ .....

108. Very lavish expenditures in railway construction in this country preceded, and therefore caused, the financial crisis of 1873.

109. The Middle States include a large territory,

New Jersey is a Middle State,

∴ .....

110. Inductive reasoning is that which proceeds from particular cases to general laws, and is eminently instrumental in the discovery of truth.

The argument that because A., B., and C. achieved eminence without a college education, and therefore a college education does not help in attaining eminence, is a case of reasoning from particular facts to a general law, and is therefore inductive, and hence conclusive.

Point out any fallacies you discover in the foregoing argument, whether in respect to the laws of Formal or of Applied Logic, especially in reference to the criteria of a valid inductive conclusion.

111. No one is free who is enslaved by his appetites,
Every sensualist is so enslaved,
∴ .....

112. David slew Goliath,
David was the sweet Psalmist of Israel,
∴ .....

113. Every liar is vile,
A vile person should be contemned,
∴ .....

114. Religion is the greatest blessing to man,
Mohammedanism is religion,
∴ .....

115. It will be clear weather if the clouds rise from the hill-tops,
The weather will not be clear,
∴ .....

116. Barns are structures for the accommodation of animals
Barns are not bird-cages,
∴ .....

117. In Opposition, prove the relation of the sub-contraries from that of the contradictories.

118. What can you get by opposition from "Benevolent actions are commendable"? By conversion from "Commendable actions are right"? By conversion from "Right actions ought to be performed"? By any mode of mediate inference from any combination of any two or more of the foregoing judgments? In every case of such combination name the kind of Syllogism resulting, and analyze it according to the laws of that kind.

119. Free-will, reason, and conscience are requisite to and constitute accountability. For wherever they are accountability is; where they are wanting there is no accountability; and as they increase in strength accountability increases.

120. Men are sensitive and rational,
∴ Brutes, being sensitive, are rational.

121. Within the last fifteen years the gold dollar has been worth two and a half times a legal-tender irredeemable paper dollar. It is now equal to such paper dollar.
∴ Gold is extremely fluctuating in value, and so unfit to be used as a measure of value.

122. Define Science, also Philosophy, and show how they differ from each other and from other kinds of knowledge.

123. Rose-water is a decoction of roses,
Rose-water is a sweet perfume,
∴ .....

Besides resolving the foregoing as a Syllogism, show which judgment is analytic and which synthetic, and why; also, whether rose-water represents a notative or symbolical conception, and why.

124. What can you infer from "Man is rational," by Opposition? "Man is accountable," by Conversion? "Man is immortal," by Reciprocal change of positive and privative conceptions?

Also from the combination of all or any two of these judgments by any form of inference, mediate or immediate?

125. Meat and drink are necessaries of life,

Meat and drink are what drunkards and gluttons spend their substance upon,

∴ .....

126. Logic is worthy to be cultivated if Aristotle is infallible,

But he is not,

∴ .....

127. A. B. is a successful demagogue,

A successful demagogue must know how to hoodwink the people,

He who knows this must understand their weaknesses,

He who understands these must lose his respect for them,

∴ .....

128. All the fish taken in the net were an indiscriminate mixture of various kinds,

Salmon and mackerel were fish that the net enclosed,

∴ .....

129. Nothing is heavier than platinum,
Feathers are heavier than nothing,
∴ Feathers are heavier than platinum.

130. If it is certain that students will be diligent, academic honors are useless;
And if it is certain they will be idle, such incitements are useless;
But one or the other of these is true;
∴ Academic honors are useless.

131. Compute the entire probability that the gentleman who died with an insurance on his life amounting to a quarter of a million of dollars, procured within the previous three months, intentionally sought his own death in order to leave this amount to his family, the probabilities of this hypothesis being—

(1) From the enormous amount insured..... ½
(2) From the insurance being effected within three months of his death..... ½
(3) From his imprudent exposure of himself to attacks of disease..... ½
(4) From his previously having lost his wife's large patrimony in speculation..... ½

The counter-probabilities being—

(1) From his unblemished character.. ..... ½
(2) And from his having carried very heavy life insurance at former periods of his life..... ½

132. Find the genus and differentia, if there be any, and give the logical character of the following definitions:

Dew is a natural deposit of water on the earth which is neither rain nor fog.

Logic is a mental science.

Conversion of Judgments is making the subject and predicate change places.

A category is a predicable. Also,

Define Logic by the method of Resolution, also of Colligation.

133. Give a Logical Division of Applied Logic, descending through the divisions and subdivisions of Fallacies. Also of Kant's categories.

134. What Inference free from all Fallacy can you get—*a*, by Opposition; *b*, by Conversion; *c*, by Mediate Inference—from

All civilized persons are educated, and

None but civilized persons are Christians,

∴ .....?

135. The tastes of men are variable, but what is according to the constitution of nature is invariable,

∴ .....

136. Reasoning by Analogy is inconclusive,

Butler's Analogy is such reasoning,

∴ .....

137. Cloven feet being found universally in horned animals, we may conclude that this fossil animal was horned, since its feet are cloven.

138. Hypotheses have often proved a fruitful means of dis-

covering and testing truth, and therefore they ought to be accepted.

239. Taking the definition of Dew above given, analyze logically the following argument, to prove that it results from the condensation of moisture in the air by contact with the earth when colder than such air:

I. This is the universal law of such contact of warm air upon colder surfaces, as is proved—

x. From the water formed on the outside of ice-pitchers,

y. Also on the surface of stones colder than the air,

z. On walls of unfurred rooms,

w. From water or ice formed on the inside of window-glass suddenly chilled.

Again, no water is deposited on surfaces as warm as the air in contact with them, as appears—

x. From pitchers or crockery as warm as the surrounding air,

y. Panes of glass when it is warm outside,

z. Stones warmed by the sun.

II. The earth at night in a clear atmosphere, such as that in which dew is formed, is made cooler than the air in contact with it by the radiation of its heat through the atmosphere in all observed instances—*x*, *y*, *z*, *w*.

Again, no dew is formed in case of the earth being as warm as the air, as is shown—

x. In cloudy nights,

y. At mid-day,

z. In decidedly cold weather.

Still further, the amount of dew formed varies with the

amount of moisture in the atmosphere and the extent to which it is warmer than the earth, as appears, because—

x. When the air becomes suddenly hot and surcharged with moisture the dews are heaviest,

y. There is little dew in time of drouth,

z. There is average dew in average clear weather.

APPENDIX B.

SYLLOGISTIC NOTATION.

1. VARIOUS methods have been adopted to represent to the eye the different forms of the Syllogism, and the relations of thought respectively involved in them. This is done through linear diagrams analogous to the figures of Geometry. It greatly assists the mind in discerning at a glance the quantity, the mutual relation, and the quality of the different terms and judgments of the syllogism, together with its figure and mood. One of the most celebrated schemes of notative symbols is that by means of circles invented by Euler, upon which we have already drawn for purposes of casual illustration. (See Chap. V. 13.)

Meaning of Syllogistic Notation.

Euler's Method by Circles.

2. Three circles are employed to denote respectively the Major, Minor and Middle Terms. Affirmative judgments are symbolized by the total or partial *in*clusion of the circle signifying the subject in that which stands for the predicate.

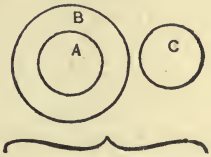
Negatives are signified by the total or partial *ex*clusion of the former from the latter.

The following diagram, in which A B and C denote respectively the minor, middle, and major terms, represents

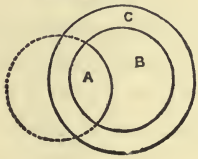
1. The moods A A A. 2. A E E. 3. A I I. 4. E I O, all of the First Figure.

1. Barbara.

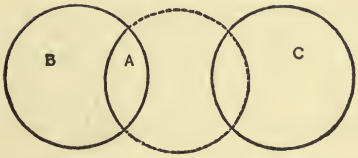
2. Celarent.



3. Darii.



4. Ferio.



Of course, this method, *mutatis mutandis*, is applicable to the other figures. This clearly and beautifully represents the Syllogism according to extension, as also the distribution or non-distribution of different terms.

NOTATION BY STRAIGHT LINES.

4. According to this scheme, a horizontal straight line denotes a term distributed. The letters S, P, or M attached, indicate that it is respectively minor, major, or middle term.

S _____
P _____

Dots are used to signify an undistributed term as noting its indefiniteness.

M .....

Any definite portion of an undistributed term is indicated by a line not dotted inserted in one that is dotted. Thus in the judgment "men are mortal," *i. e.* "some mortals," mortal is undistributed. But we take that definite portion of it which is co-extensive with the class man. Thus:

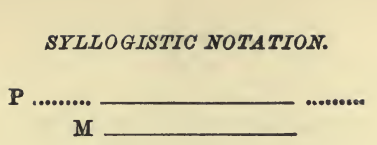
P ..... _____ ..... mortal.
S _____ men.

Affirmative judgments are symbolized by lines, one above the other—the former being the predicate, the latter the subject. Negative judgments are represented by parallel lines drawn so that one is not under the other. Thus:

P _____

S _____

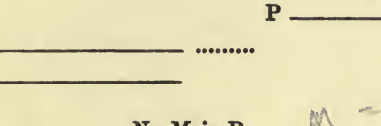
To complete the syllogism, of course three lines must be employed to represent the three terms and judgments in their quantity and other relations.



This represents A A A, of Fig. 1. Thus:

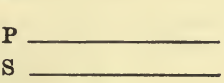
All horses are quadrupeds,
All Shetland ponies are horses,
∴ All Shetland ponies are quadrupeds.

If there be one negative premise in the Syllogism, it can be thus represented. The following is E A E, Celarent, of Fig. 1.



No M is P,
All S is M,
∴ No S is M.

Substitutive Judgments are indicated by two equal and parallel lines. Thus:



Judgments of Logical Division or Colligation (chap. II. 43) may be expressed thus:

THE HAMILTONIAN NOTATION.

5. Quite the most expressive and complete system of Notation, and one of his important contributions to Logic, is that invented by Sir William Hamilton. It is so contrived as to exhibit, at a glance, all the characteristics of the valid Syllogism, both according to intension and extension, in all the figures. This is done by means of lines, wedge-shaped in the figured Syllogism, and of uniform length and breadth in the unfigured Syllogism, and in all substitutive judgments, these latter lines denoting the perfect equality of subject and predicate.

6. The wedge-shaped figure or line denotes a judgment—its thick end the subject of extension which is contained extensively in the predicate: its thin end the subject of intension, or predicate of extension, which is contained intensively in the other. Most of what follows is so well put in Bowen's Logic, that we transfer it with little modification.

"As the employment of letters following upon each other in the same alphabet might suggest that one was invariably subordinated to the other, instead of being its subordinate in one Quantity and its superordinate in the other, Hamilton uses for the Extremes the Latin C and Greek Γ, each being the third letter in its own alphabet; as usual, M stands for Middle Term. Thus:

C ——— Γ

is read, *C and* Γ *are equal.*

C ——— Γ

may be read in two ways; Extensively, *C is included under*

ı ; Intensively, Γ *is included in C*:—or, in the usual manner, *C is* Γ, and Γ *is C*, merely remembering, without saying so, that Extension is signified in the former case, and Intension in the latter.

7. "Negation is indicated by a perpendicular stroke drawn through the line, thus: ⟝. The line without this stroke may be regarded as the Affirmative Copula; with the stroke, as the Negative Copula. A colon (:) annexed to a Term shows that it is distributed, or taken universally; a comma (,) so annexed, that it is undistributed or Particular. When a Middle Term has a colon on the right, and a comma on the left, it is understood that it is distributed when coupled in a Judgment with the Term on the right, and undistributed when coupled with the other.

8. "A line drawn beneath or above three Terms indicates the Conclusion (or the Copula of the Conclusion) deduced from the two Premises which those Terms constitute. In the Second and Third Figures, since there may be two equally direct or immediate Conclusions, they are represented by two such lines, the one above, and the other below the Premises. Thus:

C, ⟝, M : ⟝, Γ This is a Syllogism in the Second Figure, which may be read in either of the following ways.

Extensively.	*Intensively.*
Some C is some M;	All M is some Γ;
Some Γ is all M;	Some M is some C;
∴ Some Γ is some C; *or*	∴ Some C is some Γ; *or*
∴ Some C is some Γ.	∴ Some Γ is some C.

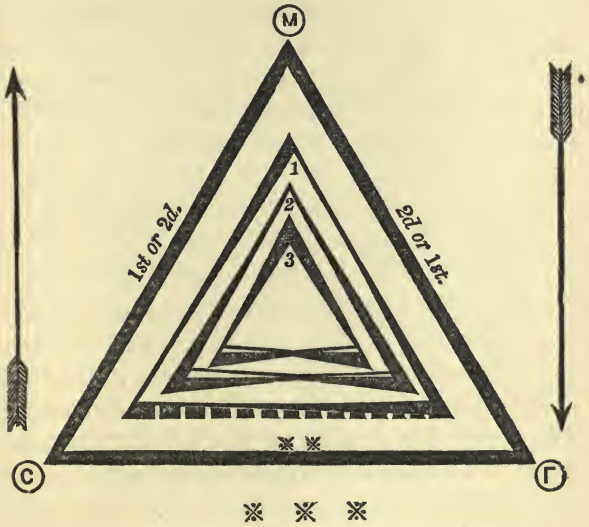
C, ——, M: —+—: r "This is a negative Syllogism in the First Figure, which may be read in either of the following ways; but in either way, it has only one *direct* or *immediate* Conclusion, though a Second Conclusion may be obtained from it *indirectly*, by converting simply the proper or direct Conclusion.

Extensively.	*Intensively.*
Some M is some C;	No M is any r;
No r is any M;	Some C is some M;
No r is some C; *or*,	Some C is not any r; *or*,
indirectly.	*indirectly.*
Some C is not any r;	Not any r is some C.

9. "The following diagram presents the whole Hamiltonian doctrine of Figure, together with the distinction between the Analytic and the Synthetic order of enouncement. After the explanations which have been given, it will be easily understood.

"As a Judgment has been designated by a line, a Syllogism, which is a union of three Judgments, is appropriately typified by a triangle, a union of three lines, of which the base represents the Conclusion, and the other two lines, the Premises. As the direction of the arrows indicates, we may proceed either in the usual or Synthetic order, from the Premises to the Conclusion, or in the reverse order, which is Analytic, from the Conclusion to the Premises. As there is no valid reason for always placing the Major Premise first in order, the diagram shows that either Premise may have precedence in this respect, so that what has been

called the Fourth Figure is here identified with the Indirect Moods of the First.

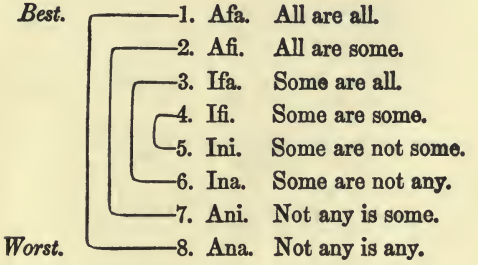


"The Unfigured Syllogism is properly represented as including all the others, as any Syllogism of either Figure may be easily expressed in this form. In like manner the triangle representing the First Figure is made to include the two typifying respectively the Second and Third, as either of the latter may be readily reduced to the former. And again, the essential unity of the Syllogistic process, and the unessential nature of variation by Figure, are appropriately signified by a single triangle comprehending all the varieties of form.

"The double Conclusions, both equally direct, in the Second and Third Figures, are shown in the crossing of two counter and corresponding lines. The Direct and Indirect Conclusions in the First Figure are distinctly typified by a common and by a broken line; the broken line is placed immediately under the other, and may thus indicate that it represents only a reflex of — a consequence through the other."

10. It will be remembered that the four fundamental judgments hitherto recognized by logicians, viz., A E I O, yield sixty-four conceivable moods. Excluding from these all that are invalid as offending against the laws of the syllogism, only eleven moods remain that are valid in the fourteen syllogisms of the first three figures, or nineteen, if the fourth figure be recognized. But Hamilton, as we have seen, recognizes eight judgments, adding to the four already named, U Y η ω. The possible combinations of these are five hundred and twelve. Of this number, however, only thirty-six will bear the tests of valid syllogisms, of which twelve are affirmamative and twenty-four negative. Thus, on this system, each affirmative mood has two corresponding ones that are negative, as each of its premises may be made negative. Since each of the moods on this system can be put in either of the three figures, there arise three times thirty-six, or one hundred and eight valid syllogisms in the several figures. The changes in the different figures, however, are for the most part unessential and insignificant. The following table by Hamilton exhibits the eight judgments recognized by him, very ingeniously in their relative strength.

in which A signifies a term distributed, I a term undistributed, f an Affirmative, and n a Negative copula. A particular is accounted weaker than a universal, and a negative weaker than an affirmative.


Best.
1. Afa. All are all.
2. Afi. All are some.
3. Ifa. Some are all.
4. Ifi. Some are some.
5. Ini. Some are not some.
6. Ina. Some are not any.
7. Ani. Not any is some.
Worst.
8. Ana. Not any is any.

"With these explanations, the following list of the twelve valid Affirmative Moods in each of the three Figures, and the twenty-four valid Negative Moods in the First Figure, all expressed in the Hamiltonian notation, will be found intelligible.

11. "In this Table, the Quantity of the Conclusion is marked only in the cases already considered, wherein the Terms obtain a different Quantity from that which they held in the Premises; accordingly, when not marked, the quantification of the Premises is held as repeated in the Conclusion. The symbol ⌣, placed beneath a Conclusion, indicates that, when the Premises are converted, the Syllogism remains in the same Mood; ⨯ shows that the two Moods between which it stands are convertible into each other by converting their Premises. The Middle Term is

THE HAMILTONIAN ANALYSIS AND SCHEME

TABLE OF SYLLO-

A. AFFIRMATIVE MOODS.

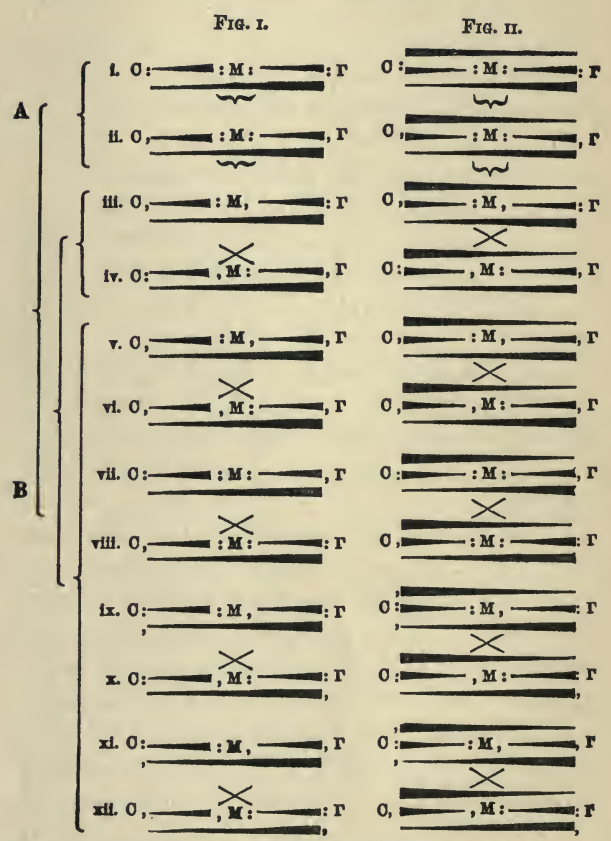


Fig. I.
Fig. II.
A
B
i. O : : M : : Γ
O : : M : : Γ
ii. O , : M : , Γ
O , : M : , Γ
iii. O , : M , : Γ
O , : M , : Γ
iv. O : , M : , Γ
O : , M : , Γ
v. O , : M , , Γ
O , : M , , Γ
vi. O , , M : , Γ
O , , M : , Γ
vii. O : : M : , Γ
O : : M : , Γ
viii. O , : M : : Γ
O , : M : : Γ
ix. O : : M , : Γ
O : : M , : Γ
x. O : , M : : Γ
O : , M : : Γ
xi. O : : M , , Γ
O : : M , , Γ
xii. O , , M : : Γ
O , , M : : Γ

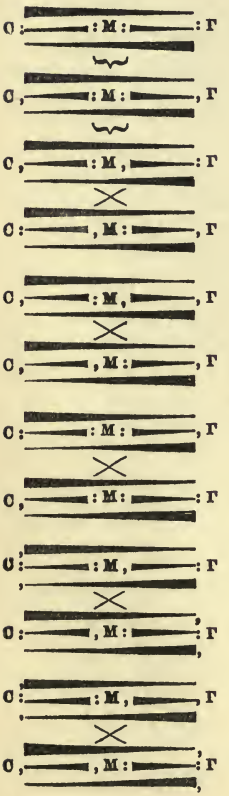
Note.—A. i. and ii. are *Balanced*. B. The other moods are *Unbalanced*. Of these,

OF NOTATION—FIGURED SYLLOGISM.

GISTIC MOODS.

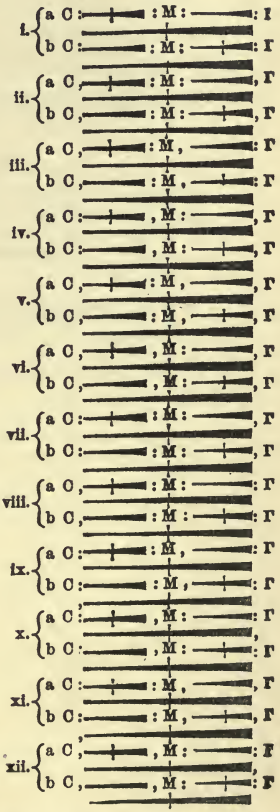
A. AFFIRMATIVE MOODS.

FIG. III.



B. NEGATIVE MOODS.

FIG. I.



iii. and iv. are unbalanced in terms only, not in propositions; the rest in both.

said to be *balanced*, when it is Universal in both Premises. The Extremes, or Terms of the Conclusion, are *balanced*, when both alike are distributed; *unbalanced*, when one is, and the other is not, distributed. Accordingly, of the Moods, in this Table, numbers I. and II. are balanced as respects both terms and propositions; in III. and IV., only the terms are unbalanced; in the remainder both terms and propositions are unbalanced."

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